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Semiconductor Measurement Technology:

HOTPAC: Programs for Thermal Analysis Including Version 3.0 of the TXYZ Program, TXYZ 30, and the Thermal MultiLayer Program, TML

John Albers

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This Special Publication is dedicated to Frank F. Oettinger
on the occasion of his retirement from Federal service.

Frank was instrumental in beginning the
NBS/NIST effort on the Kokkas model and the TXYZ codes.
He has also been supportive of the body of work presented here as well as
its integration into other Semiconductor Electronics Division activities.

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Abstract

This report presents and discusses a number of recent developments in the steady-state thermal analysis of multiple-layer structures. These include: 1) analytical evaluation of line and area average temperatures and 2) a recursion relation technique for calculating the steady-state surface temperature of a multilayer structure with an arbitrary number of layers. The application of the analytic averaging to the TXYZ code is incorporated in the updated code, TXYZ30, while the multilayer recursion relation solution along with analytic averaging are included in the Thermal MultiLayer code, TML. Both of these are contained in the HOTPAC software package.

The first part of this report contains a discussion of the general elements of the multiple-layer thermal model. This is presented in some detail for the case of the Kokkas three-layer problem and the associated TXYZ code. The previous update of the code, TXYZ20, is also discussed. This incorporates more flexible handling of input data, assignment of positive or negative noninteger weights to the various heat sources or heat sinks, and improved evaluation of limiting forms in the code.

The second part of the report presents the analytical evaluation of line and area averages which can be calculated directly. The analytical calculation of the area average temperature should provide for a more direct and convenient connection to the experimentally measured temperature which tends to average over the measurement area. The line and area averaging are incorporated into the TXYZ code to produce the TXYZ30 update.

The third portion contains a detailed discussion of the calculation of the surface temperature of a multilayer structure with an arbitrary number of layers. This is based on a recursion relation technique previously employed in electrical spreading resistance analysis to determine the surface potential from the multilayer electrical Laplace equation. The line and area averaging techniques can be applied directly to the recursion relation solution and are included in the Thermal MultiLayer, TML, code.

The appendices contain the listing of the annotated, internally documented FORTRAN source codes for TXYZ30 and TML. These codes as well as several sample input and output data files are available in ASCII format on DOS-formatted floppy disks. The sample input and output data files are included so that the user can check the programs for proper operation as well as become familiar with the setup and use of the codes. Users of the

previous versions of the TXYZ code should find the TXYZ30 and TML codes easy to use and should benefit from the wider range of problems which they can be used to address.

Key words: average temperature; FORTRAN programs; Fourier analysis; Laplace equation; multilayer model; semiconductor devices; semiconductor materials; steady-state heat flow; thermal conductivity.

PART 1. GENERAL ELEMENTS OF THERMAL MODEL

1.1 - INTRODUCTION

The operation of a semiconductor device relies on the passage of current through portions of the structure. This is accompanied by power dissipation and heating which gives rise to a temperature distribution. In time, thermal stresses may arise and lead to device degradation and possible failure. In addition to the effects of thermal stresses on device reliability, the modeling of the steady-state thermal response of a system is also useful since the temperature distribution may have an effect upon the local mechanical, thermal, electrical, or optical properties of the system. Consequently, an accurate physical model of the temperature distribution under the power condition of actual operation is of great importance. It is also possible to ascertain the relative effects of composition (thermal conductivity) and geometry (layer thickness). By carefully optimizing composition and geometry, it may be possible to minimize the thermal stress and hence ensure optimal device lifetime.

The physical and mathematical model used here is taken from the work of Kokkas which has been used previously to construct the TXYZ programs. The TXYZ computer program has been used for a number of years for the thermal analysis of semiconductor devices and packages. This program makes use of the closed form, Fourier series solution of the steady-state heat flow equation for the general case of a rectangular three-layer structure with multiple heat sources on the top surface. TXYZ provides for the calculation of the temperature at any set of points in this structure and has proven useful for the determination of the steady-state temperature distribution of semiconductor chips and packages.

The purpose of this work is to report on several recent advances which have been made in the thermal analysis. These include the analytical evaluation of line and area averages as well as the recursion relation solution of the steady-state surface temperature of a mulitlayer structure with an arbitrary number of layers.

1.2 - STEADY-STATE HEAT FLOW: SINGLE RECTANGULAR LAYER

Consider a material of uniform thermal conductivity, κ_1, in the form of a rectangular box of lateral dimensions L_x, L_y, and thickness L_1. The problem is to determine the temperature, $T(x, y, z)$, inside the material. The temperature is assumed to satisfy the steady-state heat flow equation [1]

$$\nabla^2 T(x, y, z) = 0. \tag{1}$$

As this equation is second order in the three coordinates, there are six boundary conditions. Four of these will be provided by the lateral boundary conditions. In the present problem, all four of the lateral boundary conditions are provided by the assumption that the lateral surfaces are adiabatic; i.e., there is no heat flow out of the lateral boundaries of the material;

$$\left.\frac{\partial T(x,y,z)}{\partial x}\right|_{x=0,L_x} = \left.\frac{\partial T(x,y,z)}{\partial y}\right|_{y=0,L_y} = 0. \tag{2}$$

The remaining two boundary conditions will be provided by the vertical boundary conditions (in z). These vertical boundary conditions will not be specified at the present time as the intent of the present section is to obtain a general solution of the one-layer problem where only the lateral boundary conditions are specified. As the above equation is formulated in Cartesian coordinates, it is convenient to use Fourier analysis techniques to solve the x and y portion of the equation. The Fourier transform with respect to the variables x and y is used, remembering that the geometry is constrained to $0, L_x$, and $0, L_y$. This is defined as [2]

$$\tau(f_x, f_y, z) = \int_0^{L_x} \int_0^{L_y} T(x,y,z) \exp\big(-2\pi i(xf_x + yf_y)\big) dx dy, \tag{3}$$

where f_x, f_y are the Fourier transform variables which are conjugate to the variables x, y. The inverse Fourier transform is defined as

$$T(x,y,z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau(f_x, f_y, z) \exp\big(2\pi i(xf_x + yf_y)\big) df_x df_y. \tag{4}$$

The requirement that there is no heat flow out of the sides of the structure, i.e., $\partial T(x,y,z)/\partial x$ and $\partial T(x,y,z)/\partial y$ are zero when x and y are either equal to zero or to L_x or L_y, respectively, leads to a consideration of the expression

$$\frac{\partial T(x,y,z)}{\partial x} =$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau(f_x, f_y, z) \exp(2\pi i y f_y) 2\pi f_x \Big\{ -\sin(2\pi f_x x) + i\cos(2\pi f_x x) \Big\} df_x df_y, \tag{5}$$

and a similar expression for $\partial T(x,y,z)/\partial y$. If this is to be zero at the origin, this would require that the cosine term be removed. Next, consider the resulting expression at the other lateral boundary, i.e., at $x = L_x$ where it is supposed to be zero. The only way that this could be the case is if the argument of the sine function is an integer times π, or that the Fourier transform variable is of the form

$$f_x = \frac{n}{2L_x}. \tag{6}$$

The same argument applies to the y-dependent portion. The Fourier representation of the temperature with the above lateral boundary conditions may then be written as

$$T(x,y,z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau(n,m,z) \cos(n\pi x/L_x) \cos(m\pi y/L_y) df_x df_y. \tag{7}$$

The Fourier transform equation is now written as

$$\tau(n,m,z) = \int_0^{L_x} \int_0^{L_y} T(x,y,z) \cos(n\pi x/L_x) \cos(m\pi y/L_y) dx dy. \tag{8}$$

As the system is of finite size, it is convenient to write the integral in eq (7) as a sum over the Fourier cosine terms which fit into the rectangular geometry. Further, as the cosine function is symmetric around the origin, the sums may be written over only the positive values of the indices. It is important to keep in mind that the terms corresponding to $m, n{=}0$ do not have the factor of 2 coming from the symmetry of the cos function. In addition, the differentials may be written as

$$df_x = \Delta \frac{n}{L_x} = \frac{n+1}{L_x} - \frac{n}{L_x} = \frac{1}{L_x}. \tag{9}$$

The Fourier representation of the temperature may be written as

$$T(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{4\tau(n,m,z) \cos(n\pi x/L_x) \cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1) L_x L_y}, \tag{10}$$

where $\delta_{nn'}$ is the Kronecker delta and is equal to unity if $n = n'$ and zero otherwise. By substituting eq (10) into eq (1) and using

$$\nabla^2 = \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\}, \tag{11}$$

and

$$\frac{\partial^2}{\partial x^2} \cos(n\pi x/L_x) = -(n\pi/L_x)^2 \cos(n\pi x/L_x), \tag{12}$$

and the same relation for the y-dependence, it is straightforward to show that

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{4\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_n+1)(\delta_m+1)L_xL_y}\left\{-(n\pi/L_x)^2-(m\pi/L_y)^2+\frac{\partial^2}{\partial z^2}\right\}\tau(n,m,z)=0. \tag{13}$$

As the sum is zero for arbitrary values of the variables x and y (and the cosine terms are, in general, nonzero), then a necessary and sufficient condition that eq (13) is satisfied is that

$$\left\{-(n\pi/L_x)^2-(m\pi/L_y)^2+\frac{\partial^2}{\partial z^2}\right\}\tau(n,m,z)=0. \tag{14}$$

This differential equation may be solved analytically using elementary methods. If the variable, γ, is defined as

$$\gamma=\left\{\left(\frac{n\pi}{L_x}\right)^2+\left(\frac{m\pi}{L_y}\right)^2\right\}^{1/2}, \tag{15}$$

eq (14) may be rewritten as

$$\frac{\partial^2}{\partial z^2}\tau(n,m,z)-\gamma^2\tau(n,m,z)=0. \tag{16}$$

The solution of this equation is

$$\tau(n,m,z)=\alpha\cosh(\gamma z)+\beta\sinh(\gamma z), \tag{17}$$

where the coefficients α and β, which may be functions of γ, are determined from the two z-dependent boundary conditions.

The above equation is the general solution for the z-dependent Fourier expansion coefficients for a single rectangular layer. It provides a convenient basis set for the discussion of the problem of a rectangular multilayer structure where all of the layers have the same lateral dimensions. For the multilayer case, the solution in each of the layers can be expressed in the form of the above equation where the coefficients are to be determined from the two z-dependent boundary conditions appropriate to each of the layers. This is used in the next section where the multilayer problem is discussed and then specialized to the three-layer case.

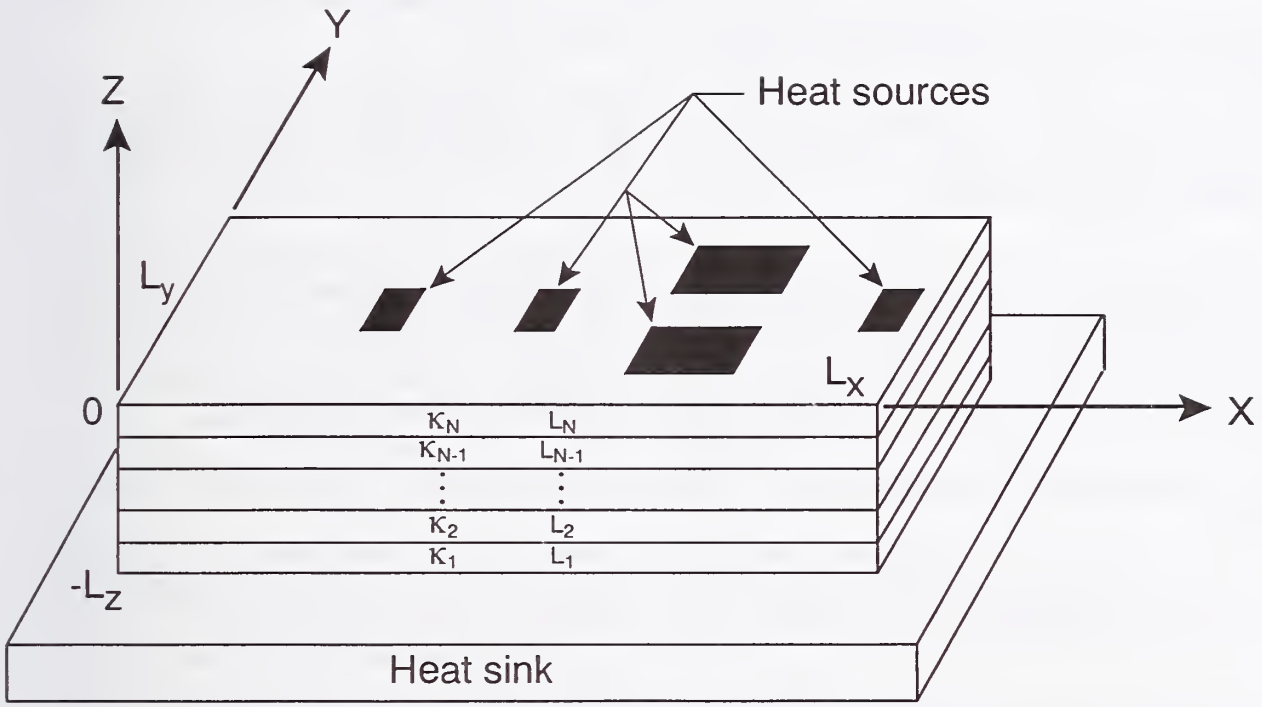

Figure 1. Geometry of the multilayer structure in which the steady-state temperature is calculated.

1.3 - STEADY-STATE HEAT FLOW: MULTILAYER RECTANGULAR STRUCTURE

The basic problem considered in this section is the general approach to the calculation of the temperature in a multilayer structure depicted in figure 1. The layers are characterized in terms of the thermal conductivities and thicknesses $\kappa_i, L_i (i = 1, 2, \ldots, N)$, where the numbering begins at the bottom layer and proceeds up to the top or N-th layer. Then, the bottom layer will be layer number 1 with temperature $T_1(x, y, z)$. The next layer up will be layer number 2 with temperature $T_2(x, y, z)$. This continues to the top layer which will be layer N with temperature $T_N(x, y, z)$.

The equations are discussed in general for the N-layer problem and then specialized to the three-layer problem as originally described by Kokkas [3,4].

This will allow for the discussion of the Kokkas three-layer solution and the TXYZ code while providing the necessary development for the discussion of the recursion relation in the third part of the report.

The thicknesses and thermal conductivities of these layers are of considerable importance in the dissipation of heat generated by the power sources on the surface of the top layer. These power sources are typically the regions at or near the surface of the device where currents are passed into the device during normal operation. Consequently, the generation of heat in the device is one of the unavoidable side effects of device operation.

The mathematical formulation of this problem is based upon the following set of assumptions:

1) the lateral dimensions of all the layers in the structure are equal while the thicknesses may be different;

2) each layer is of uniform, isotropic, temperature-independent thermal conductivity;

3) there is no heat loss from the lateral surfaces due to either radiation or convection (adiabatic surfaces) - heat flow in the structure takes place by conduction;

4) there is no input power density inside the structure - heat is generated only on the top surface;

5) there are no heat losses due to interconnections to the top layer; and

6) the heat sink, which is in contact with the bottom layer, is ideal and has a temperature equal to ambient.

The temperature in each of the layers is assumed to satisfy the steady-state heat flow equation,

$$\nabla^2 T(x, y, z) = 0. \tag{18}$$

The general one-layer solution provides a convenient and useful basis for the multilayer solution. Then, the temperature in each layer may be written as

$$T_i(x,y,z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{4\tau_i(n,m,z)\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1)L_xL_y}. \tag{19}$$

The Fourier coefficients for the solution of the steady-state heat flow equation in the i-th layer of an N-layer structure may be written as

$$\tau_i(n,m,z) = \tau_i(\gamma z) = \alpha_i \cosh(\gamma z) + \beta_i \sinh(\gamma z), (i = 1, ..., N), \tag{20}$$

where the expansion coefficients, α_i and β_i, are determined from the z-dependent boundary conditions.

Notice that the assumption that there is no heat flow out of the lateral boundaries,

$$\left.\frac{\partial T_i(x,y,z)}{\partial x}\right|_{x=0,L_x} = \left.\frac{\partial T_i(x,y,z)}{\partial y}\right|_{y=0,L_y} = 0, \quad (i = 1, 2, \ldots, N), \tag{21}$$

is already contained in eq (19) as discussed in the previous single-layer analysis.

The 2N z-dependent boundary conditions used to solve the set of equations in eqs (19) and (20) may be expressed as follows.

First, heat enters (or leaves) the structure through the portions of the top layer where power is applied. This is expressed as

$$\kappa_N \left.\frac{\partial T_N(x,y,z)}{\partial z}\right|_{z=0} = P(x,y), \tag{22}$$

where κ_N is the thermal conductivity of the top layer, and where $P(x,y)$ is the power function. This is expressed as $P(x,y) = P_0U(x,y)$, where P_0 is the power density (W/cm^2), and $U(x,y)$ describes the surface geometrical distribution of the power sources. Next, the bottom-layer boundary condition is provided by the requirement that the temperature is continuous across the interface between the bottom layer and the heat sink and equal to the ambient which is taken as zero,

$$T_1(x,y,z) = T_a = 0, \tag{23}$$

where the function is evaluated at the interface. Finally, the remaining $2(N-1)$ conditions are provided by the requirement that the temperature and the normal component of the heat flow are continuous across the internal interfaces. These are expressed as

$$T_i(x,y,z) = T_{i-1}(x,y,z), \tag{24}$$

$$\kappa_i \frac{\partial T_i(x,y,z)}{\partial z} = \kappa_{i-1} \frac{\partial T_{i-1}(x,y,z)}{\partial z}, \tag{25}$$

where the functions and their derivatives are to be evaluated at the interfacial boundaries.

For the case of an N-layer structure, the substitution of eqs (19) and (20) into the boundary conditions given by eqs (22 - 25) gives rise to a set of $2N$ equations in $2N$ unknowns ($\{\alpha_i(\gamma)\}$, $\{\beta_i(\gamma)\}$, $i = 1, ..., N$). The analytic solution of this system of equations requires the use of matrix algebra involving Cramer's rule [5] and the Laplace method [5] for the evaluation of the resulting determinants. Clearly, this also can become rather tedious, especially since the expansion coefficients are functions of the continuous variable, γ.

For cases up to $i = 3$, Kokkas [3,4] was able to work out the system of equations for not only the surface temperature but also the temperature inside the three-layer structure. The Kokkas three-layer solution is reviewed in the next section and the N-layer problem is returned to in Part 3.

1.4 - KOKKAS THREE-LAYER MODEL

The Kokkas three-layer model [3,4] involves the solution of the multilayer equations, eqs (19, 20, 22 - 25), for the case where N=3. This can be carried out analytically as the systems of equations are still tractable. Specializing the multilayer equations to the case of three layers yields the Fourier coefficients

$$\tau_i(n,m,z) = \tau_i(\gamma z) = \alpha_i \cosh(\gamma z) + \beta_i \sinh(\gamma z), (i = 1,2,3). \tag{26}$$

The corresponding boundary conditions are

$$\kappa_3 \frac{\partial T_N(x,y,z)}{\partial z}\bigg|_{z=0} = P(x,y), \tag{27}$$

$$T_3(x,y,z)\Big|_{z=-L_3} = T_2(x,y,z)\Big|_{z=-L_3}, \tag{28}$$

$$T_2(x,y,z)\Big|_{z=-(L_3+L_2)} = T_1(x,y,z)\Big|_{z=-(L_3+L_2)}, \tag{29}$$

$$\kappa_3 \frac{\partial T_3(x,y,z)}{\partial z}\bigg|_{z=-L_3} = \kappa_2 \frac{\partial T_2(x,y,z)}{\partial z}\bigg|_{z=-L_3}, \tag{30}$$

$$\kappa_2 \frac{\partial T_2(x,y,z)}{\partial z}\bigg|_{z=-(L_3+L_2)} = \kappa_1 \frac{\partial T_1(x,y,z)}{\partial z}\bigg|_{z=-(L_3+L_2)}. \tag{31}$$

$$T_1(x,y,z)\Big|_{z=-L_z} = T_a = 0, \tag{32}$$

where all temperatures are measured relative to the ambient heat sink temperature and $L_z = L_3 + L_2 + L_1$.

It is important to note that the origin of the depth scale is at the surface of the top layer and that all vertical distances are negative.

The solution of the system of equations is by standard matrix algebra techniques. However, instead of using this method explicitly here, the Fourier coefficients for each of the layers are presented and are shown to satisfy the heat flow equation and the appropriate boundary conditions. In particular, the Fourier coefficients in the three layers are presented in reference [4]. Specializing these to the steady-state situation, they are

$$\tau_3(n,m,z) = A\Big\{B\cosh\big(\gamma(L_3+z)\big) + C\sinh\big(\gamma(L_3+z)\big)\Big\}, \tag{33}$$

$$\tau_2(n,m,z) = A\Big\{D\cosh\big(\gamma(L_3+L_2+z)\big) + E\sinh\big(\gamma(L_3+L_2+z)\big)\Big\}, \tag{34}$$

$$\tau_1(n,m,z) = A\sinh\big(\gamma(L_3+L_2+L_1+z)\big), \tag{35}$$

where

$$A = \frac{U(n,m)P_0}{\kappa_3\gamma}\left\{\frac{1}{B\sinh(\gamma L_3) + C\cosh(\gamma L_3)}\right\}, \tag{36}$$

$$B = D\cosh(\gamma L_2) + E\sinh(\gamma L_2), \tag{37}$$

$$C = \frac{\kappa_2}{\kappa_3}\Big\{D\sinh(\gamma L_2) + E\cosh(\gamma L_2)\Big\}, \tag{38}$$

$$D = \sinh(\gamma L_1), \tag{39}$$

$$E = \frac{\kappa_1}{\kappa_2} \cosh(\gamma L_1), \tag{40}$$

and

$$U(n,m) = \int_0^{L_x} \int_0^{L_y} U(x,y) \cos(n\pi x/L_x) \cos(m\pi y/L_y) dx dy, \tag{41}$$

is the double Fourier cosine transform of the power density uniformity function.

Now, it is shown that the above are the solutions of the z-dependent part of the steady-state heat flow equation (see eq (16))

$$\frac{\partial^2}{\partial z^2} \tau_i(n,m,z) - \gamma^2 \tau_i(n,m,z) = 0, \tag{42}$$

where the subscript i takes on the values of 3,2,1. This may be easily shown to be the case as

$$\frac{\partial^2}{\partial z^2} \cosh\big(\gamma(L+z)\big) = \gamma^2 \cosh\big(\gamma(L+z)\big), \tag{43}$$

and

$$\frac{\partial^2}{\partial z^2} \sinh\big(\gamma(L+z)\big) = \gamma^2 \sinh\big(\gamma(L+z)\big), \tag{44}$$

where L is a constant and is equal to L_3, $L_3 + L_2$, or $L_3 + L_2 + L_1$ in eqs (33 - 35). Hence, eqs (33 - 35) satisfy the z-dependent differential equation. Next, it is shown that these Fourier coefficients satisfy the appropriate boundary conditions. The first of these is that

$$\kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\bigg|_{z=0} = P(n,m) = U(n,m) P_0. \tag{45}$$

Using the Fourier coefficient given by eq (33) for the top layer, this may be evaluated as

$$\kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\bigg|_{z=0} = \kappa_3 \gamma A \Big\{ B\sinh(\gamma L_3) + C\cosh(\gamma L_3) \Big\}$$

$$= \kappa_3 \gamma \frac{U(n,m)P_0}{\kappa_3 \gamma} \left\{ \frac{1}{B\sinh(\gamma L_3) + C\cosh(\gamma L_3)} \right\} \Big\{ B\sinh(\gamma L_3) + C\cosh(\gamma L_3) \Big\}$$

$$= U(n,m)P_0 . \tag{46}$$

Hence, the top-layer boundary condition is satisfied by the $\tau_3(n,m,z)$. Next, consider the bottom-layer boundary condition, i.e.,

$$\tau_1(n,m,z)\Big|_{z=-(L_3+L_2+L_1)} = 0. \tag{47}$$

Making use of eq (35), this may be readily evaluated as

$$\tau_1(n,m,z)\Big|_{z=-(L_3+L_2+L_1)} = A\sinh\big(\gamma(L_3+L_2+L_1-L_3-L_2-L_1)\big) = 0. \tag{48}$$

Hence, the last boundary condition is satisfied. The final set of boundary conditions to be verified are the ones which pertain to the interface boundary conditions. These are

$$\tau_3(n,m,z)\Big|_{z=-L_3} = \tau_2(n,m,z)\Big|_{z=-L_3}, \tag{49}$$

$$\tau_2(n,m,z)\Big|_{z=-(L_3+L_2)} = \tau_1(n,m,z)\Big|_{z=-(L_3+L_2)}, \tag{50}$$

$$\kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\bigg|_{z=-L_3} = \kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\bigg|_{z=-L_3}, \tag{51}$$

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\bigg|_{z=-(L_3+L_2)} = \kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\bigg|_{z=-(L_3+L_2)}. \tag{52}$$

In order to verify these equations, it is simplest to calculate the right- and left-hand sides of the equations and then compare them directly. The left-hand side of the first temperature continuity equation may be evaluated as

$$\tau_3(n,m,z)\Big|_{z=-L_3} = A\Big\{B\cosh\big(\gamma(L_3-L_3)\big)+C\sinh\big(\gamma(L_3-L_3)\big)\Big\}$$
$$= AB = A\Big\{D\cosh(\gamma L_2)+E\sinh(\gamma L_2)\Big\}. \tag{53}$$

The right-hand side of the equation may be evaluated as

$$\tau_2(n,m,z)\Big|_{z=-L_3} = A\Big\{D\cosh\big(\gamma(L_3+L_2-L_3)\big)+E\sinh\big(\gamma(L_3+L_2-L_3)\big)\Big\}$$
$$= A\Big\{D\cosh(\gamma L_2)+E\sinh(\gamma L_2)\Big\}. \tag{54}$$

Hence, the first of the temperature continuity equations satisfies the boundary condition.

Next, consider the second temperature continuity equation,

$$\tau_2(n,m,z)\Big|_{z=-(L_3+L_2)} = \tau_1(n,m,z)\Big|_{z=-(L_3+L_2)}. \tag{55}$$

The left-hand side of the equation may be evaluated as

$$\tau_2(n,m,z)\Big|_{z=-(L_3+L_2)} = A\Big\{D\cosh\big(\gamma(L_3+L_2-L_3-L_2)\big)+E\sinh\big(\gamma(L_3+L_2-L_3-L_2)\big)\Big\}$$
$$= AD = A\sinh(\gamma L_1). \tag{56}$$

The right-hand side is

$$\tau_1(n,m,z)\Big|_{z=-(L_3+L_2)} = A\sinh\big(\gamma(L_3+L_2+L_1-L_3-L_2)\big) = A\sinh(\gamma L_1). \tag{57}$$

Hence, the second temperature continuity equation is satisifed. The next equation to be verified is the first heat-flow continuity equation; i.e.,

$$\kappa_3\frac{\partial\tau_3(n,m,z)}{\partial z}\Big|_{z=-L_3} = \kappa_2\frac{\partial\tau_2(n,m,z)}{\partial z}\Big|_{z=-L_3}. \tag{58}$$

Evaluating the left-hand side leads to

$$\kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\bigg|_{z=-L_3} = \kappa_3 \gamma A\Big\{B\sinh\big(\gamma(L_3-L_3)\big) + C\cosh\big(\gamma(L_3-L_3)\big)\Big\}$$

$$= \kappa_3 \gamma AC = \kappa_2 \gamma A\Big\{D\sinh(\gamma L_2) + E\cosh(\gamma L_2)\Big\}. \tag{59}$$

The right-hand side may be evaluated as

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\bigg|_{z=-L_3} = \kappa_2 \gamma A\Big\{D\sinh(\gamma L_2) + E\cosh(\gamma L_2)\Big\}. \tag{60}$$

The final equation to be evaluated is that for the heat flow continuity between the second and third layers; i.e.,

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\bigg|_{z=-(L_3+L_2)} = \kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\bigg|_{z=-(L_3+L_2)}. \tag{61}$$

The left-hand side of this equation may be evaluated as

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\bigg|_{z=-(L_3+L_2)}$$

$$= \kappa_2 \gamma A\Big\{D\sinh\big(\gamma(L_3+L_2-L_3-L_2)\big) + E\cosh\big(\gamma(L_3+L_2-L_3-L_2)\big)\Big\}$$

$$= \kappa_2 \gamma AE = \kappa_1 \gamma A\cosh(\gamma L_1). \tag{62}$$

The right-hand side is

$$\kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\bigg|_{z=-(L_3+L_2)} = \kappa_1 \gamma A\cosh\big(\gamma(L_3+L_2+L_1-L_3-L_2) = \kappa_1 \gamma A\cosh(\gamma L_1). \tag{63}$$

Now that it has been shown that the Fourier coefficients satisfy the steady-state heat flow problem and the appropriate boundary conditions, there are several points to be considered before getting into the body of the program. These include: (1) evaluation of the function $U(n,m)$ for a uniform power source of given size and (2) simplification of the Fourier coefficients for subsequent numerical analysis. The latter point is necessary as the limits of small γ and large γ may give rise to overflow or underflow problems when the program is constructed.

1.5 - KOKKAS THREE-LAYER MODEL: FORM OF THE FUNCTION U(n,m)

The first thing to be considered is the form of the function $U(n,m)$ for an arbitrary number of heat sources. This function is defined as

$$U(n,m) = \int_0^{L_x} \int_0^{L_y} U(x,y)\cos(n\pi x/L_x)\cos(m\pi y/L_y)dxdy. \tag{64}$$

The analysis can most easily be accomplished in terms of a single uniform heat source. The case of an arbitrary number of heat sources follows by summing the results of each heat source. Further, if any of the heat sources are nonuniform, their effects can be constructed by suitably overlapping a number of uniform heat sources. In the coordinate system being used, consider a single heat source denoted by the index i with a corner at the location (x_i, y_i) and lengths along the x- and y-directions given by (lx_i, ly_i). Over the area of the heat source, $U(x,y)$ is assumed to be uniform and equal to unity. Away from the area of the heat source, $U(x,y)$ is assumed to be equal to zero. Then, $U(x,y)$ may be viewed as being a unit step function over the surface of the power source. Consequently, the contribution from this single heat source may be written as

$$\int_0^{L_x} \int_0^{L_y} U(x,y)\cos(n\pi x/L_x)\cos(m\pi y/L_y)dxdy =$$
$$\int_{x_i}^{x_i+lx_i} \int_{y_i}^{y_i+ly_i} \cos(n\pi x/L_x)\cos(m\pi y/L_y)dxdy. \tag{65}$$

The integrals can be simply evaluated to give the result that

$$U_i(n,m) = \frac{L_x L_y}{(n\pi)(m\pi)}\left\{\sin\left(\frac{n\pi(x_i+lx_i)}{L_x}\right) - \sin\left(\frac{n\pi x_i}{L_x}\right)\right\}$$
$$\times\left\{\sin\left(\frac{m\pi(y_i+ly_i)}{L_y}\right) - \sin\left(\frac{m\pi y_i}{L_y}\right)\right\}. \tag{66}$$

Similar expressions may be written for each of the heat sources and then summed to give the cumulative heat source effect.

Before turning to the small γ and large γ behavior of the Fourier coefficients, it is important to consider the behavior of the function $U(n,m)$ for either $n = 0$, $m = 0$, or both. This is important in the numerical implementation of the solutions as the program will have to calculate the double Fourier cosine transform over the range of n, m required by the sum in eq (19). Once the value of n or m is zero, there will be problems with most machines as far as evaluating the seeming divergence. This can be circumvented by investigating the

behavior of the function for $n = 0$. This may be readily carried out by first noting that the function $U(n, m)$ (for the particular form of $U(x, y)$) is a product of two terms. This may be simply written as $U(n, m) = U(n)U(m)$. Then, consider the n (from the x integration) contribution to the function which is given by

$$U(n) = \frac{L_x}{n\pi}\left\{\sin\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \sin\left(\frac{n\pi x_i}{L_x}\right)\right\}. \tag{67}$$

There is an apparent divergence or infinity if n is simply set equal to zero. This is the way in which a computer would look at the expression. However, this infinity is not real as can be seen by using the expansion of the sine function for small values of the argument. In particular,

$$\sin(x) = x - \frac{x^3}{3!} + \cdots. \tag{68}$$

Making use of this expression for the sine function, it is straightforward to show that

$$\lim_{n\to 0} U(n) = \lim_{n\to 0} \frac{L_x}{n\pi}\left\{\sin\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \sin\left(\frac{n\pi x_i}{L_x}\right)\right\} =$$
$$\lim_{n\to 0} \frac{L_x}{n\pi}\left\{\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \left(\frac{n\pi x_i}{L_x}\right)\right\} = lx_i. \tag{69}$$

The same conclusion holds for the y-dependent portion, i.e., $U(m)$. This must be specially coded to bypass any overflow problem. The specific coding of the heat source may be found in the program listing in the function UZERO.

In general, the function $U(n, m)$ is oscillatory and does not approach zero sufficiently fast for large values of n or m. In particular, eq (66) shows that $U_i(n, m) \to 0$ like $1/nm$ as $m, n \to \infty$. In addition, the cosine terms in eq (19), i.e., $\cos(n\pi x/L_x)\cos(m\pi y/L_y)$, do not approach a definite limit for large values of the argument. Consequently, the product $U(n, m)\cos(n\pi x/L_x)\cos(m\pi y/L_y)$ tends to zero slowly.

1.6 - KOKKAS THREE-LAYER MODEL: BEHAVIOR OF FOURIER COEFFICIENTS FOR SMALL VALUES OF THE ARGUMENT

As has been seen in the treatment of the function $U(n, m)$, care must be taken for the case where both n and m are equal to zero (or $\gamma = 0$). The same considerations must be carried out for the Fourier coefficients in the three layers. In the solutions in the three layers, the summation indices n, m appear. The summation over these variables is of influence in the variable γ according to eq (15). Also, the Fourier coefficients contain the hyperbolic functions which depend upon γ. For large values of γ, the sinh and cosh functions grow

exponentially. This can present special numerical problems when the summation variables approach the upper limits which may be required for the case of very small heat sources. Hence, special care must be taken to study the behavior of the Fourier coefficients for small γ and large γ to remove any potential numerical overflow problems. Once this is properly taken care of, the Fourier coefficients and the solutions will be numerically well behaved.

First, consider the small γ behavior of the Fourier coefficients. This is done by considering the small γ behavior of the eqs (33 - 40) and the small argument behavior of the hyperbolic functions. In the following discussion as well as the discussion of the large γ behavior, the term $U(n,m)P_0/\kappa_3$ will be removed for convenience. This term will henceforth be included explicitly in the sum in eq (10) as the Fourier coefficients for all three layers contain this as a common factor through A (see eqs (33 - 36)). Then, for small γ,

$$E \approx \frac{\kappa_1}{\kappa_2}, \tag{70}$$

$$D \approx \gamma L_1, \tag{71}$$

$$C \approx \frac{\kappa_2}{\kappa_3}\left\{\gamma^2 L_2 L_1 + \frac{\kappa_1}{\kappa_2}\right\}, \tag{72}$$

$$B \approx \gamma L_1 + \frac{\kappa_1}{\kappa_2}\gamma L_2, \tag{73}$$

and, remembering that the factor $U(n,m)P_0/\kappa_3$ has been included explicitely in eq (19),

$$A \approx \frac{1}{\gamma}\frac{\kappa_3}{\kappa_1}. \tag{74}$$

Making use of these expressions and the small argument behavior of the hyperbolic functions, it is straightforward to investigate the small γ behavior of the solutions. In particular,

$$\tau_3(n,m,z) \approx \frac{1}{\gamma}\frac{\kappa_3}{\kappa_1}\left\{\gamma L_1 + \frac{\kappa_1}{\kappa_2}\gamma L_2 + \frac{\kappa_2}{\kappa_3}\left\{\gamma^2 L_2 L_1 + \frac{\kappa_1}{\kappa_2}\right\}\gamma(L_3+z)\right\}$$

$$\approx \frac{\kappa_3}{\kappa_1}\left\{L_1 + \frac{\kappa_1}{\kappa_2}L_2 + \frac{\kappa_1}{\kappa_3}(L_3+z)\right\}.$$

Then,

$$\lim_{\gamma \to 0} \tau_3(n,m,z) = (L_3 + z) + \frac{\kappa_3}{\kappa_2} L_2 + \frac{\kappa_3}{\kappa_1} L_1. \tag{75}$$

Next,

$$\tau_2(n,m,z) \approx \frac{1}{\gamma} \frac{\kappa_3}{\kappa_1} \Big\{ \gamma L_1 + \frac{\kappa_1}{\kappa_2} \gamma (L_3 + L_2 + z) \Big\}.$$

Then,

$$\lim_{\gamma \to 0} \tau_2(n,m,z) = \frac{\kappa_3}{\kappa_1} L_1 + \frac{\kappa_3}{\kappa_2} (L_3 + L_2 + z). \tag{76}$$

And finally,

$$\tau_1(n,m,z) \approx \frac{1}{\gamma} \frac{\kappa_3}{\kappa_1} \Big\{ \gamma (L_3 + L_2 + L_1 + z) \Big\}.$$

Then,

$$\lim_{\gamma \to 0} \tau_1(n,m,z) = \frac{\kappa_3}{\kappa_1} (L_3 + L_2 + L_1 + z). \tag{77}$$

These special forms of the Fourier coefficients (in the limit as $\gamma \to 0$) are necessary in the code to bypass overflow problems for small values of the argument.

1.7 - KOKKAS THREE-LAYER MODEL: BEHAVIOR OF FOURIER COEFFICIENTS FOR LARGE VALUES OF THE ARGUMENT

As the Fourier coefficients have been investigated for small γ and have been shown to be well-behaved when properly written, what remains is to write these coefficients in a form which is amenable for investigating their large γ behavior. As noted before, the hyperbolic functions, sinh and cosh, grow exponentially for large values of the argument. On the other hand, the hyperbolic tanh approaches unity for large values of the argument. With this in mind, let us investigate the form of the Fourier coefficients, written as much as possible in terms of the tanh, which takes care of this potential numerical difficulty. To this end, it is convenient to introduce the shorthand notation for the hyperbolic functions, $c(x) = \cosh(x)$, $s(x) = \sinh(x)$, and $t(x) = \tanh(x)$. Making use of this shorthand notation, the Fourier coefficients may be written as

$$\tau_3(n,m,z) = A \Big\{ Bc\big(\gamma(L_3 + z)\big) + Cs\big(\gamma(L_3 + z)\big) \Big\}, \tag{78}$$

$$\tau_2(n,m,z) = A\Big\{Dc\big(\gamma(L_3+L_2+z)\big) + Es\big(\gamma(L_3+L_2+z)\big)\Big\}, \tag{79}$$

$$\tau_1(n,m,z) = As(\gamma(L_3+L_2+L_1+z), \tag{80}$$

where,

$$A = \frac{1}{\gamma}\left\{\frac{1}{Bs(\gamma L_3) + Cc(\gamma L_3)}\right\}, \tag{81}$$

$$B = Dc(\gamma L_2) + Es(\gamma L_2), \tag{82}$$

$$C = \frac{\kappa_2}{\kappa_3}\Big\{Ds(\gamma L_2) + Ec(\gamma L_2)\Big\}, \tag{83}$$

$$D = s(\gamma L_1), \tag{84}$$

$$E = \frac{\kappa_1}{\kappa_2}c(\gamma L_1). \tag{85}$$

As in the investigation of the small γ behavior of the Fourier coefficients, the factor $U(n,m)P_0/\kappa_3$ has been deleted from eq (81) for convenience. This factor may simply be included in the Fourier representation of the temperature, eq (19), as it is common to all three layers.

Now, the above equations (eqs (78 - 85)) will be rewritten by making use of the definition of the hyperbolic tanh; i.e., $t(x) = s(x)/c(x)$. First, consider the coefficient C.

$$C = \frac{\kappa_2}{\kappa_3}\Big\{Ds(\gamma L_2) + Ec(\gamma L_2)\Big\},$$

$$C = \frac{\kappa_2}{\kappa_3}\left\{s(\gamma L_1)s(\gamma L_2) + \frac{\kappa_1}{\kappa_2}c(\gamma L_1)c(\gamma L_2)\right\},$$

$$C = \frac{\kappa_2}{\kappa_3}c(\gamma L_1)c(\gamma L_2)\left\{t(\gamma L_1)t(\gamma L_2) + \frac{\kappa_1}{\kappa_2}\right\}. \tag{86}$$

Next, the coefficient B may be written as

$$B = Dc(\gamma L_2) + Es(\gamma L_2),$$

$$B = s(\gamma L_1)c(\gamma L_2) + \frac{\kappa_1}{\kappa_2}c(\gamma L_1)s(\gamma L_2),$$

$$B = c(\gamma L_1)c(\gamma L_2)\left\{t(\gamma L_1) + \frac{\kappa_1}{\kappa_2}t(\gamma L_2)\right\}. \tag{87}$$

Then,

$$Bs(\gamma L_3) + Cc(\gamma L_3) =$$

$$c(\gamma L_1)c(\gamma L_2)s(\gamma L_3)\left\{t(\gamma L_1) + \frac{\kappa_1}{\kappa_2}t(\gamma L_2)\right\} + c(\gamma L_1)c(\gamma L_2)c(\gamma L_3)\frac{\kappa_2}{\kappa_3}\left\{t(\gamma L_1)t(\gamma L_2) + \frac{\kappa_1}{\kappa_2}\right\}$$

$$= c(\gamma L_1)c(\gamma L_2)c(\gamma L_3)\left\{t(\gamma L_1)t(\gamma L_3) + \frac{\kappa_1}{\kappa_2}t(\gamma L_3)t(\gamma L_2) + \frac{\kappa_2}{\kappa_3}t(\gamma L_1)t(\gamma L_2) + \frac{\kappa_1}{\kappa_3}\right\}. \tag{88}$$

Then, the coefficient A may be written as

$$A =$$

$$\frac{1}{\gamma c(\gamma L_1)c(\gamma L_2)c(\gamma L_3)}\frac{1}{\left\{t(\gamma L_1)t(\gamma L_3) + \frac{\kappa_1}{\kappa_2}t(\gamma L_3)t(\gamma L_2) + \frac{\kappa_2}{\kappa_3}t(\gamma L_1)t(\gamma L_2) + \frac{\kappa_1}{\kappa_3}\right\}}. \tag{89}$$

It is convenient to define the function $\Omega(\gamma)$ as

$$\Omega(\gamma) = \frac{1}{\left\{t(\gamma L_1)t(\gamma L_3) + \frac{\kappa_1}{\kappa_2}t(\gamma L_3)t(\gamma L_2) + \frac{\kappa_2}{\kappa_3}t(\gamma L_1)t(\gamma L_2) + \frac{\kappa_1}{\kappa_3}\right\}}, \tag{90}$$

which is well behaved for all values of γ. Then, the coefficient A may be written as

$$A = \frac{\Omega(\gamma)}{\gamma c(\gamma L_1)c(\gamma L_2)c(\gamma L_3)}. \tag{91}$$

By making use of the above procedure, it is relatively straightforward to show that the Fourier coefficients as given by eqs (78 - 80) may be written as

$$\tau_3(n,m,z) =$$

$$\frac{\Omega(\gamma)c\big(\gamma(L_3+z)\big)}{\gamma c(\gamma L_3)}\left\{t(\gamma L_1)+\frac{\kappa_1}{\kappa_2}t(\gamma L_2)+t\big(\gamma(L_3+z)\big)\frac{\kappa_2}{\kappa_3}\left(t(\gamma L_1)t(\gamma L_2)+\frac{\kappa_1}{\kappa_2}\right)\right\}, \quad (92)$$

$$\tau_2(n,m,z) = \frac{\Omega(\gamma)c\big(\gamma(L_3+L_2+z)\big)}{\gamma c(\gamma L_3)c(\gamma L_2)}\left\{t(\gamma L_1)+\frac{\kappa_1}{\kappa_2}t\big(\gamma(L_3+L_2+z)\big)\right\}, \quad (93)$$

and

$$\tau_1(n,m,z) = \frac{\Omega(\gamma)s\big(\gamma(L_3+L_2+L_1+z)\big)}{\gamma c(\gamma L_3)c(\gamma L_2)c(\gamma L_1)}. \quad (94)$$

In eqs (92 - 94), the function $\Omega(\gamma)$ and the terms inside the curly brackets are well behaved for all values of the variable γ. The sinh and cosh terms which remain may still give rise to numerical overflow problems for large values of the argument. However, as both of these functions grow exponentially for large values of the argument and they appear in both the numerator and the denominator of the Fourier coefficients, there will be cancellation. This cancellation for large values of the argument will not be worked out in detail here but is contained in the FORTRAN listing of the program in the function FUNZ.

These Fourier coefficients are used in the equation

$$T_i(x,y,z) = P_0\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{4U(n,m)\tau_i(n,m,z)\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1)L_xL_y\kappa_3} \quad (95)$$

for $i = 1,2,3$ to obtain the solutions in each of the three layers. In the above, the term $P_0U(n,m)/\kappa_3$ has been written out explicitly and is no longer contained in the Fourier coefficients. It is important in obtaining the solution in x,y,z to use the appropriate layer equation. This is automatically taken care of in the program as the depth z is compared with the various thicknesses and the corresponding layer Fourier coefficient is used. The user does not have to specify which layer is to be used.

There are several parts of the above equation which can be identified. First, there is the z-dependent part of the solution in each layer, $\tau_i(n,m,z)$. The two cosine functions, $\cos(n\pi x/L_x)\cos(m\pi y/L_y)$, arise from the x- and y-dependent parts of the steady-state equation. Their specific form arises from the requirement of the lateral boundary conditions. Finally, the requirement that heat enters or leaves the top surface only through the

heating elements introduces the function $P_0 U(n,m)$. It should be noted that P_0 appears only in front of the sum. It is a scale or multiplicative factor which for convenience may be set equal to unity. Its effect is to scale the temperature in a direct linear fashion. Performing the calculation for $P_0 = 1$ for a given structure then provides the temperature distribution for arbitrary P_0 which can then be obtained by multiplying by the P_0 which is used.

An interesting exercise left to the reader is to show when the three thermal conductivities are equal that the one-layer solution is obtained. Also, another exercise is to show that the one-layer solution is obtained when the thicknesses of the second and third layers are set equal to zero.

1.8 - SPECIAL CASE OF POWER SOURCE COVERING TOP SURFACE

As a special case of eq (95), consider the situation of a single power source completely covering the top surface; i.e., there is a single heat source with lateral dimensions equal to that of the three-layer structure. In this particular case, it is shown that the above equation reduces to the familiar thermal resistance equation. The easiest way to proceed with the analysis is to consider the specific form of the function $U(n,m)$. From eq (66),

$$U_1(n,m) =$$
$$\frac{L_x L_y}{(n\pi)(m\pi)}\left\{\sin\left(\frac{n\pi(x_1+lx_1)}{L_x}\right)-\sin\left(\frac{n\pi x_1}{L_x}\right)\right\}\left\{\sin\left(\frac{m\pi(y_1+ly_1)}{L_y}\right)-\sin\left(\frac{m\pi y_1}{L_y}\right)\right\}. \quad (96)$$

For the particular case of uniform surface coverage, $x_1 = y_1 = 0$, $lx_1 = L_x$, and $ly_1 = L_y$. Upon substituting these values into the equation, the function reduces to

$$U_1(n,m) = \frac{L_x L_y}{(n\pi)(m\pi)}\left\{\sin(n\pi)\sin(m\pi)\right\}. \quad (97)$$

This is zero when the indices are nonzero. For the case where both of the indices are zero, the use of the expansion of the sine function gives rise to the result that

$$U_1(n,m) = L_x L_y \delta_{n0}\delta_{m0}. \quad (98)$$

If this form of the $U(n,m)$ function is substituted into eq (95) for the temperature in each of the three layers and the form of the Fourier expansion coefficients as $\gamma \to 0$ (eqs (75 - 77)) is used, it is readily shown that the temperatures in the three layers may be written as

$$T_3(x,y,z) = P_0\left\{\frac{L_3+z}{\kappa_3}+\frac{L_2}{\kappa_2}+\frac{L_1}{\kappa_1}\right\}, \quad (99)$$

$$T_2(x,y,z) = P_0\left\{\frac{L_3 + L_2 + z}{\kappa_2} + \frac{L_1}{\kappa_1}\right\}, \tag{100}$$

$$T_1(x,y,z) = P_0\left\{\frac{L_3 + L_2 + L_1 + z}{\kappa_1}\right\}. \tag{101}$$

As P_0 is the power density per unit area, these equations give rise to the usual results of the one-dimensional calculations of the thermal resistance.

1.9 - EFFECT OF UPPER SUMMATION LIMITS ON TEMPERATURE

It is important to keep in mind that the Fourier coefficients are functions of the variables n and m. As discussed in the previous sections, the power density function, $U(n,m)$, tends to zero very slowly for large values of the argument. Also, the cosine terms in eq (95) do not tend to any limit as the arguments approach infinity. It is the Fourier coefficients which are responsible for the convergence of the sum. This is especially the case for the surface ($z = 0$) value of the temperature. This consideration comes into play when setting the upper summation limit.

Consider the case of a structure with lateral dimension of L and a heat element of lateral dimension of Δ. For the Fourier series to begin "seeing" this element, the cosine function in the basis set must have at least one complete cycle in Δ. As the element is rectangular, one cycle is certainly not enough, as the cosine is a poor representation of the rectangle. Consequently, higher "frequencies" are required to assure the adequacy of the representation. The rule of thumb is that the number of terms should be at least on the order of L/Δ. A stronger rule of thumb would require several times L/Δ. In general, the stronger rule of thumb should be applied to the smallest heat element to achieve better accuracy.

1.10 - THE TXYZ AND TXYZ20 CODES

The above equations have been coded in the TXYZ program [6]. Over time, various updates were included in the TXYZ20 program [7]. In the original version of TXYZ, all heat sources had equal unit weights. The updated TXYZ20 program allows for noninteger weights to be assigned to each heating element. This weight may be positive or negative and hence represent the effects of either a heat source or a heat sink.

The reason for the relatively compact nature of the original TXYZ code and its numerical efficiency is that great care was exercised in the investigation of the functions for the small and large argument regimes. For small values of the argument, the evaluation of limiting forms led to expressions which were correct and free of artificial numerical singularities. For large values of the argument, special care was required. Many of the functions involved contained the hyperbolic sinh and cosh functions. These approach the exponential function for large values of the argument. These provide the possibility for numerical infinities or overflows. Careful investigation of the forms of the functions showed that these numerical

infinities were removable by other numerical infinities. For example, an exponential in the numerator and denominator of a function would give rise to two overflows when evaluated by a computer but would cancel each other and give rise to a finite result when analytic evaluation was properly applied. The construction of the function, FUNZ, in the TXYZ code was based upon such evaluations and their numerical implementations. With the passage of time and the use of the TXYZ code, one place where such cancellation was found to be incomplete was in the case of a fairly thick middle layer. For most cases of interest to the modeling of the steady-state thermal response of semiconductor device structures, these layers are usually thin. However, this situation has now been carefully investigated and the numerically stable limiting form has been determined and used in the code in the middle-layer portion of the FUNZ function. The introduction of the nonsingular limiting form is contained in the TXYZ20 code.

The original TXYZ program and the TXYZ20 update have been used for a number of investigations. Some of these include: the thermal evaluation of VLSI packages using test chips [8], the understanding of thermal resistance measurements [9], the investigation of the thermal interaction between electromigation test structures [10], and the benchmarking of other codes as, for example, in the modeling of MMIC devices for the determination of the channel temperatures during life tests [11]. The codes have been run on a variety of machines. They were originally programmed and run on a VAX 11/785. They now are run on PCs and Sun SPARC10 Workstations [12].

PART 2. ANALYTIC EVALUATION OF LINE AND AREA AVERAGES

2.1 - INTRODUCTION

Electrical and optical measurements are often used to determine the local steady-state temperature of semiconductor device structures. For example, the electrical resistance of a segment of a metallization stripe may be used to measure the average temperature of that segment of the stripe [13]. Measurements of the temperature of a metal line on a thin layer of silicon dioxide deposited on a silicon substrate have been performed previously. These have been used in conjunction with the calculation of the average temperature of the metal line in order to determine the thermal conductivity of the silicon dioxide layer [14].

Another example is found in an electrical technique for the measurement of the peak junction temperature of power transistors [15]. This has recently been applied to the use of the gate voltage in the measurement of the average channel temperature of a power GaAs MESFET (MEtal-Semiconductor Field Effect Transistor) [16,17]. The work in reference [5] employs the calculated average channel temperature and the measured average channel temperature to determine a scale factor which is used to extract the peak channel temperature. It is the peak channel temperature which plays a central role in establishing operating conditions for reliable device performance.

The temperature calculations involved in the above determinations of the thermal conductivity of a SiO_2 film [14] and the peak channel temperature of power GaAs MESFETs [17]

make use of the Kokkas model for the steady-state temperature in a rectangular device structure. In particular, the point function temperature, $T(x, y, z)$, was calculated by using the numerical implementation of the Kokkas model [4] as presented in the TXYZ codes [6,7]. However, as indicated in the above discussion, the measured temperature is actually an average over the area over which the electrical measurement is performed. In order to make the connection between the measured average temperature and the calculated average temperature, references [14] and [17] use a set of assumed representative points at which $T(x, y, z)$ is calculated. These values are then used to construct what might be called a point-by-point evaluation of the average.

A number of questions need to be addressed in this point-by-point evaluation of the average. First, for a given heat source or active area geometry, how does one choose a set of representative points? Second, how many points are sufficient to adequately represent the effects of variations of $T(x, y, z)$ over the area? Third, what are the effects of any residual oscillations in $T(x, y, z)$ which are inherent in the Fourier analysis involved in the model?

Fortunately, the Kokkas model and the TXYZ numerical implementation provide for an analytic evaluation of the averages over arbitrary areas as well as line segments. This gives the averages directly and thus obviates consideration of questions related to the choice of a set of representative points as well as the number of points. In addition, the calculation yields a uniformly convergent value of the average, thus bypassing any residual oscillations of individual values of $T(x, y, z)$. The averages take less time to compute than the point-by-point evaluations and provide a better representation of the measured averages.

2.2 - CALCULATION OF AVERAGE TEMPERATURES

From eq (95), the point function temperature, $T_i(x, y, z)$, is

$$T_i(x, y, z) = P_0 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{4U(n, m)\tau_i(n, m, z)\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_{n0} + 1)(\delta_{m0} + 1)L_x L_y \kappa_3}. \tag{102}$$

As indicated above, the point-by-point evaluation of the area average temperature makes use of a series of point values given by eq (102).

Alternatively, the average temperatures may be calculated directly. The development applies to both line and area averages. Line averages provide a certain amount of information, but it is the area average which is related to the experimentally measured temperature. Consider an arbitrary line segment from (x_j) to $(x_j + lx_j)$ and an arbitrary rectangular area from (x_j, y_j) to $(x_j + lx_j, y_j + ly_j)$. These are pictured in figure 2 as well as in the top view shown in figure 3. It is important to note that these are shown on the surface $(z = 0)$ of the structure for the purposes of clarity and illustration. The arbitrary line segment and arbitrary area (in the x-y plane) may be located on or inside the structure $(0 \leq z \leq -(L_1 + L_2 + L_3))$.

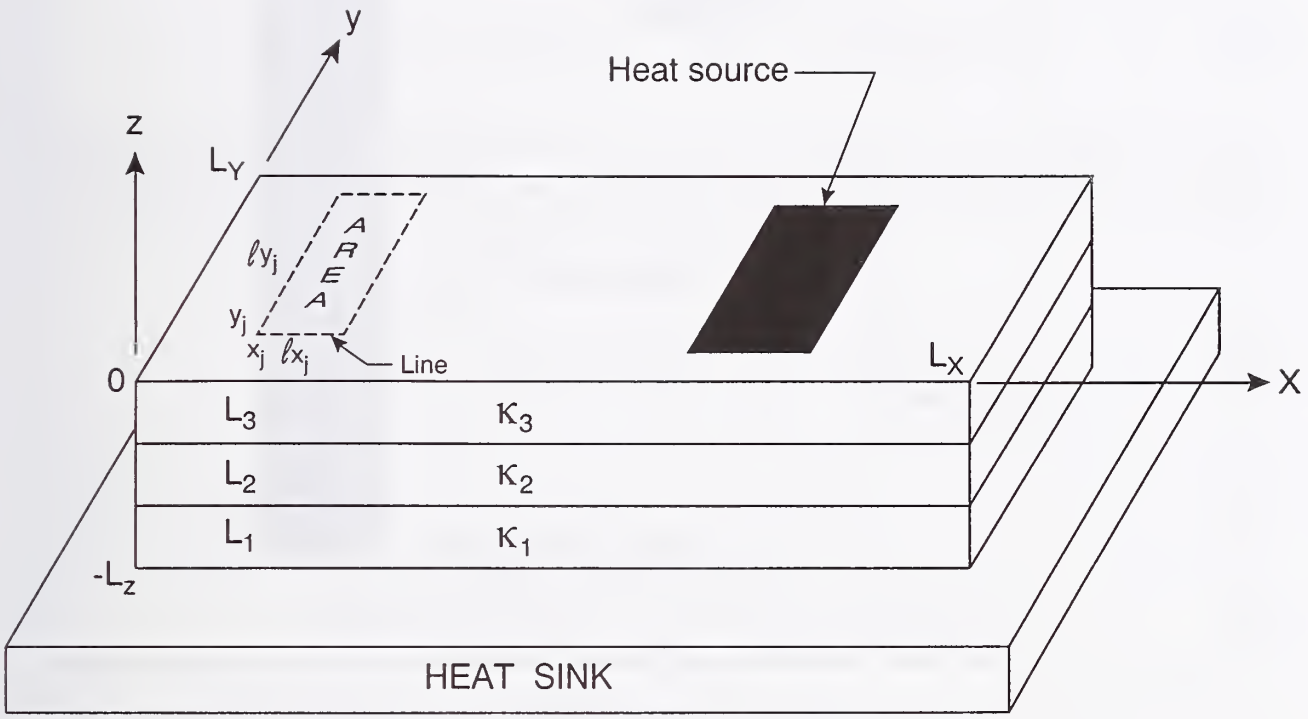

Figure 2. Geometry of the structure in which the line average or area average temperature is calculated.

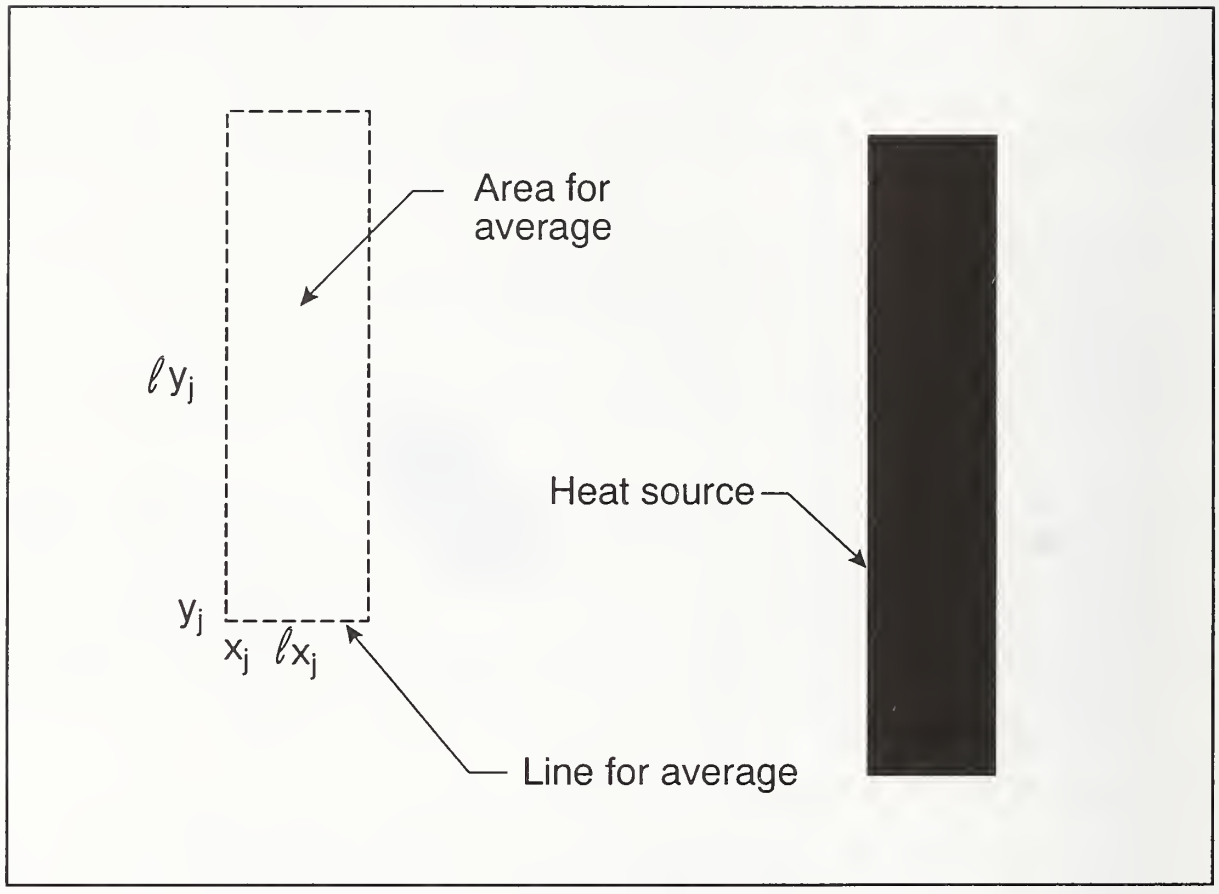

Figure 3. Top view of the surface of the structure shown in figure 2. The line for averaging and area for averaging are separated from the heat source for the purpose of clarity.

The z-dependence is explicitly expressed in terms of the averages which are defined below. The average of $T_i(x,y,z)$ over the arbitrary line segment is defined as

$$\langle T_i(y,z)\rangle_l = \frac{1}{lx_j}\int_{x_j}^{x_j+lx_j} T_i(x,y,z)dx, \tag{103}$$

and the average of $T_i(x,y,z)$ over the arbitrary rectangular area

$$\langle T_i(z)\rangle_a = \frac{1}{lx_j\cdot ly_j}\int_{x_j}^{x_j+lx_j}\int_{y_j}^{y_j+ly_j} T_i(x,y,z)dxdy. \tag{104}$$

Notice that the x and y dependence of $T_i(x,y,z)$ is completely explicit. This means that the integrals in the line and area averages may be analytically evaluated. It is convenient to define the functions, $\Lambda_x(n)$ and $\Lambda_y(m)$, by

$$\Lambda_x(n) = \frac{L_x}{n\pi}\left\{\sin\Big(\frac{n\pi(x_j+lx_j)}{L_x}\Big) - \sin\Big(\frac{n\pi x_j}{L_x}\Big)\right\}, \tag{105}$$

and

$$\Lambda_y(m) = \frac{L_y}{m\pi}\left\{\sin\Big(\frac{m\pi(y_j+ly_j)}{L_y}\Big) - \sin\Big(\frac{m\pi y_j}{L_y}\Big)\right\}. \tag{106}$$

By substituting eq (102) into eqs (103) and (104) and making use of the definitions of the functions, $\Lambda_x(n)$ and $\Lambda_y(m)$, given by eqs (105) and (106), it is straightforward to evaluate the line and area averages.

Then, the average of $T_i(x,y,z)$ over the arbitrary line segment is

$$\langle T_i(y,z)\rangle_l = \frac{P_0}{lx_j}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{4U(n,m)\tau_i(n,m,z)\Lambda_x(n)\cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1)L_xL_y\kappa_3}, \tag{107}$$

and the average of $T_i(x,y,z)$ over the arbitrary rectangular area is

$$\langle T_i(z)\rangle_a = \frac{P_0}{lx_j\cdot ly_j}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{4U(n,m)\tau_i(n,m,z)\Lambda_x(n)\Lambda_y(m)}{(\delta_{n0}+1)(\delta_{m0}+1)L_xL_y\kappa_3}. \tag{108}$$

Several observations about the line and area average temperature defined by eqs (107) and (108) are important to note. First, the averages may be calculated by a single evaluation of the double sum. This means that the averages may be calculated about as quickly as the evaluation of the point value of the temperature, $T_i(x, y, z)$. Second, the averages may include any part or all of the x-y plane defined by the structure. If the area is on the top surface, it also may contain none, part, or all of the heat source(s). This flexibility provides for the evaluation of area average temperatures over a portion of the heat source or over a portion of the surface away from the heat source(s). Third, the averages may be calculated for the surface ($z = 0$) as well as any depth ($z < 0$) inside the structure. Area averages calculated for different areas and as a function of depth provide information on the temperature spreading and heat transport inside the structure.

2.3 - NUMERICAL IMPLEMENTATION AND TXYZ30

The basic TXYZ program has been used as the framework in which to construct the programs to numerically evaluate the equations for the line and area averages in eqs (107) and (108). The resulting FORTRAN77 code has been named TXYZ30. This code has been constructed to allow for the calculation of point functions, line averages, or area averages by means of an integer variable switch which is set in the beginning of the input data file.

It is important to keep in mind that the lines and areas over which the averages are computed are arbitrary. They may be on the top surface ($z = 0$) or inside the structure ($z < 0$). In addition, the averages may be easily computed not only over part or all of the heat sources but also at locations away from the heat sources. Most applications of these codes would probably make use of area averaging. Line averaging may be useful in examining the line average temperatures. Typical evaluations of the averages take about the same CPU time as is required to calculate a single point value of $T(x, y, z)$.

The direct calculation of the area average temperature vastly simplifies the analysis of the data when investigating the thermal conductivity of thin surface layers. The direct calculation of the averages provides for a faster and more accurate technique for the case of the determination of the peak channel temperature from the ratio of the measured average channel temperature and the calculated average channel temperature. This is especially relevant when there may be a significant variation of $T(x, y, z)$, both across and along the active area. The point-by-point evaluation of the average channel temperature must take this into account to provide an adequate representation of the variation to ensure suitable accuracy. The use of eq (108) is not encumbered by this requirement and provides the averages directly.

The TXYZ30 code is contained in the first appendix. The FORTRAN source code is also contained in the HOTPAC software package. In addition, there are several sample files. These are txyz30io.1, txyz30io.2 and txyz30io.3 where the io refers to input/output. The file extension refers to the example of a point function (1), a line average (2) and an area average (3). These files contain the input data, the annotated input data and the corresponding output data for each case. The user can electronically cut out the input

data for the input.dat file needed to run TXYZ30.

2.4 - TOTAL AREA AVERAGES AND THE THERMAL RESISTANCE

An interesting example of the use of eq (108) is provided by the situation where the average is performed over the entire top surface ($z = 0$) area of the structure. For the case of a single heat source of dimensions lx by ly, it is straightforward to show that the average over the total area (average surface temperature) is

$$\langle T(0)\rangle_a = P_0 \frac{lx \cdot ly}{L_x L_y} \sum_{i=1}^{3} \frac{L_i}{\kappa_i}, \tag{109}$$

which is just the one-dimensional (thermal resistance) result scaled by the ratio of the heat source area and the total area.

2.5 - APPLICATIONS OF AREA AVERAGES

The evaluation of the averages is simple and direct and easily performed for any number of arbitrary line segments or areas. The computed averages over part or all of the heat source area should give a more direct representation of the measured values and hence provide a more accurate determination of the thermal properties as well as peak temperatures. In addition, the averages are not restricted to the areas where heat enters the structure (over part or all of the heat source or sources). The averages may also be computed for various depths inside the structure. Both nonheat source averages and depth averages can provide information on the effects of structure geometry and thermal properties on the heat flow.

Infrared imaging techniques are also used to experimentally measure the surface temperature of semiconductor device structures. The temperature is an average over the spot size of the imager. Application of the calculation of the area averages for this situation will speed the analysis of the average temperatures being measured. This should substantially improve the understanding and application of the techniques.

Before turning to the thermal multilayer problem, it is important to notice that the application of the averaging technique is general and depends upon the explicit x- and y-dependence of the temperature expressed in eq (102). The thermal multilayer problem will have exactly the same kind of explicit x- and y-dependence so that the averaging can be applied directly. This is discussed in the next part of the report.

PART 3. RECURSION RELATION SOLUTION OF MULTILAYER MODEL

3.1 - INTRODUCTION

It has been two decades since Kokkas' original work on the thermal analysis of multiple-layer structures [3,4]. This is based on the solution of the heat flow equation for a multilayer rectangular structure. While the interface boundary conditions for the continuity of the

temperature and the heat flow are for a general multilayer structure, the equations have been solved in detail and evaluated for the case of up to a three-layer structure. The results of the three-layer steady-state heat flow analysis are exact for this situation. This three-layer solution has been numerically implemented in the TXYZ programs [3,4]. This model and the codes have found wide use in a variety of steady-state problems.

During this same 20-year time span, there have been a number of advances which have taken place in the theoretical analysis of the analogous electrical problem of two-probe spreading resistance. The starting point of the work on the electrical problem is the multilayer Laplace equation analysis of Schumann and Gardner [18,19]. Originally, the numerical analysis required a mainframe computer to do the rather cumbersome matrix algebra which develops quickly once the calculations get past two or three layers. This limitation coupled with difficulties involved in the evaluation of the accompanying oscillatory integrals served to severely limit the applicability of the technique. However, a major advance came with the introduction of a recursion relation by Choo and coworkers [20] for the construction of the solution of the N-layer problem from the solution of the $(N-1)$-layer problem. The recursion relation had previously been introduced in the geophysical literature [21,22]. The book by Koefoed [21] provides a comprehensive discussion of Laplace's equation and its solution, the multilayer approach applied to geoelectric resistance measurements, and the derivation of the recursion relation. It should be noted that the mathematical description of spreading resistance and geoelectric resistance are exactly the same even though the length scales are vastly different.

The introduction of the recursion relation eliminated the cumbersome matrix manipulations and allowed for the relatively straightforward algebraic evaluation of the functions required for the multilayer analysis. The form of the recursion relation has been clarified by Berkowitz and Lux [23] and has provided one of the cornerstones of the routine calculation of the spreading resistance from the resistivity structure as well as the extraction of the resistivity structure from the spreading resistance. The remaining cornerstone was introduced by inventive quadrature techniques which allows for the rapid evaluation of the necessary integrals. Calculations which would have been prohibitive if not impossible 20 years ago can now be performed routinely on PC-based systems. The model, the numerical analysis, and the FORTRAN codes have been summarized recently [24].

As the electrical problem and the thermal problem are similar from the mathematical viewpoint, the question arose as to the applicability of the recursion relation technique, with possible and appropriate modification, to the steady-state thermal problem. If this could be accomplished, then it would provide for an exact solution of the steady-state surface temperature for a multilayer structure with *any* number of layers. The present work describes the modified recursion relation and its application to the thermal problem. The salient features of the recursion relation are presented along with a comparison of its results with those of the Kokkas three-layer solution. The use of the recursion relation for extension of the calculations beyond three layers is discussed, with emphasis on the small increase in calculation time for each added layer. The computational efficiency of the recursion relation is also discussed. This new technique should prove extremely useful

in the understanding of the thermal effects of multilayer structures.

As the similarity to the spreading resistance problem is part of the motivation, a brief discussion of the analysis involved in this problem is presented. This is followed by the corresponding analysis for the multilayer thermal problem. The connection of these two problems is then made, along with a discussion of the need for a modified recursion relation for the thermal case. Its form is then determined and the thermal recursion relation is presented. Finally, the implications of this thermal recursion relation are discussed.

3.2 - THE N-LAYER ELECTRICAL PROBLEM

For the case of the spreading resistance problem, the multilayer solution of the Laplace equation provides for the calculation of the resistance between the contacts on the top surface of a nonuniform conductivity (resistivity) structure. The fundamental assumption of the multilayer analysis is that Laplace's equation is satisfied in each "layer" in the material. The problem is set up in cylindrical coordinates to emulate the symmetry of a single circular contact on the top surface of the material. The potential is assumed to be independent of the angular variable in this coordinate system. The Laplace equation may then be written as

$$\nabla^2 V(r,z) = \frac{\partial^2}{\partial r^2} V(r,z) + \frac{1}{r}\frac{\partial}{\partial r} V(r,z) + \frac{\partial^2}{\partial z^2} V(r,z) = 0, \tag{110}$$

where $V(r,z)$ is the potential, r is the radial coordinate, and z is the depth coordinate. This equation may be solved by means of separation of variables, with the result that a particular solution is

$$V(r,z) = \exp(-\lambda z) J_0(\lambda r) + \exp(+\lambda z) J_0(\lambda r), \tag{111}$$

where $J_0(\lambda r)$ is the Bessel function and λ^2 is the separation of variables constant. The boundary condition on the r part of the solution is that $V(r,z)$ approaches zero as r tends to infinity. This is satisfied by the above for all values of λ. Then, the general solution is an integral of the particular solution with a weighting factor and is of the form

$$V(r,z) = \int_0^\infty \Big\{ \big(1 + \theta(\lambda)\big) \exp(-\lambda z) + \psi(\lambda) \exp(+\lambda z) \Big\} J_0(\lambda r) d\lambda, \tag{112}$$

where the weighting functions, $\theta(\lambda)$ and $\psi(\lambda)$, are determined from the z-dependent boundary conditions. The above is the general solution of Laplace's equation in cylindrical coordinates for a single layer. This provides the framework in which the multilayer solution may be addressed.

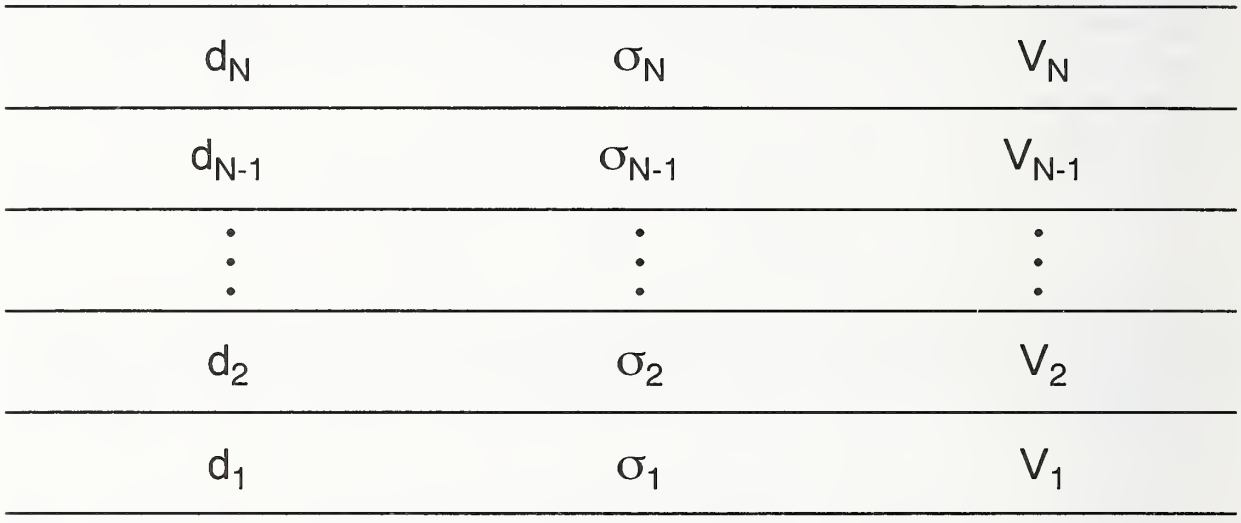

Figure 4. Schematic representation of the geometry used in the Schumann and Gardner multilayer Laplace equation analysis. In this figure, d_i, σ_i, and V_i are the thickness, electrical conductivity, and potential, respectively, in the i-th layer.

The depth-dependent portion of the multilayer geometry is presented in figure 4. Each layer is described by a thickness and electrical conductivity. Charge neutrality is assumed in each layer so that the potential in each layer satisfies Laplace's equation. The general one-layer solution given by eq (112) provides a convenient and useful basis for the required layer solution. Then, the solution of Laplace's equation in the i-th layer in an N-layer structure may be written as

$$V_i(r,z) = \int_0^\infty \Big\{ \big(1+\theta_i(\lambda)\big)\exp(-\lambda z) + \psi_i(\lambda)\exp(+\lambda z) \Big\} J_0(\lambda r) d\lambda. \tag{113}$$

The boundary conditions used to solve the system of equations (i.e., determine $\{\theta_i(\lambda)\}$ and $\{\psi_i(\lambda)\}$, $i = 1, ..., N$) are provided by conditions on the top surface, the intermediate interfaces, and the bottom surface. On the top surface, current flow takes place only through the probe which is modeled as a circular plate of radius a. Then the top surface boundary condition is expressed as

$$-\sigma_N \frac{\partial V_N(r,z)}{\partial z} = J(r), \tag{114}$$

where σ_N is the electrical conductivity of the top layer, $J(r)$ is the probe current density [18,19], and the derivative is evaluated at $z = 0$. On the bottom surface, the potential is assumed to be well behaved and, more specifically, is assumed to approach zero. This is usually expressed as

$$\lim_{z\to\infty} V_1(r,z) = 0. \tag{115}$$

Equations (114) and (115) provide 2 of the $2N$ conditions required to solve the N-layer system of equations. The remaining $2(N-1)$ conditions are provided by requirements at the interfaces between the layers where the potentials and the current densities are assumed to be continuous [18,19]. These are expressed as

$$V_i(r,z) = V_{i-1}(r,z), \tag{116}$$

and

$$\sigma_i \frac{\partial V_i(r,z)}{\partial z} = \sigma_{i-1} \frac{\partial V_{i-1}(r,z)}{\partial z}, \tag{117}$$

where the functions and their derivatives are to be evaluated at the interfacial boundaries.

For the case of an N-layer structure, the substitution of eq (113) into the boundary conditions given by eqs (114 - 117) gives rise to a set of $2N$ equations in $2N$ unknowns ($\{\theta_i(\lambda)\}$, $\{\psi_i(\lambda)\}$, $i = 1, ..., N$). The analytic solution of this system of equations requires the use of the Cramer's rule of linear algebra. Clearly, this can become rather tedious especially since the expansion coefficients are functions of the continuous variable, λ. It is possible to show that the potential on the top surface of the N-layer structure may be written as [18,19]

$$V_N(r,0) = \frac{I}{2\pi a \sigma_N} \int_0^\infty \frac{A_N(\lambda) \sin(\lambda a) J_0(\lambda r)}{\lambda} d\lambda, \tag{118}$$

where the kernel function, $A_N(\lambda) = 1 + 2\theta_N(\lambda)$, depends upon the electrical conductivities and thicknesses of the "layers" in the multilayer structure (through the solution of the above system of simultaneous equations). Schumann and Gardner were able to work out the system of equations for the cases up to $i = 3$. Cases beyond three layers presented a major stumbling block.

The development of the recursion relation by Koefoed [21] and its use by Choo *et al.* [20] provided a major breakthrough in the evaluation of eq (118) and effectively removed the numerical difficulty associated with matrix inversion. The philosophy and utility of a recursion relation is that the kernel function, $A_N(\lambda)$, for an N-layer structure can be easily generated from the kernel of an $(N-1)$ layer structure by means of an *algebraic* relation. This algebraic relation represents a reduction of the matrix algebra and the subsequent manipulations with determinants. The reader may wish to refer to Koefoed's book for a detailed discussion. In particular, if the kernel is known for the $(N-1)$ layer case, then the kernel for the N layer case is given by

$$A_N(\lambda) = \frac{\sigma_{N-1}\omega(\lambda) + \sigma_N A_{N-1}(\lambda)}{\sigma_{N-1} + \sigma_N \omega(\lambda) A_{N-1}(\lambda)}, \tag{119}$$

where

$$\omega(\lambda) = \frac{1 - \exp(-2\lambda d_N)}{1 + \exp(-2\lambda d_N)}, \tag{120}$$

and d_N is the thickness of layer N. In practice, the recursion relation is begun with the one-layer case from which the two-layer kernel is generated. Then the three-layer kernel is generated from the two-layer kernel. This sequence is repeated until the N-layer kernel is determined. Notice how the conductivity and thickness of the N-th layer enter in the recursion relation.

Berkowitz and Lux [23] have shown that the recursion relation for the kernel function could be expressed as

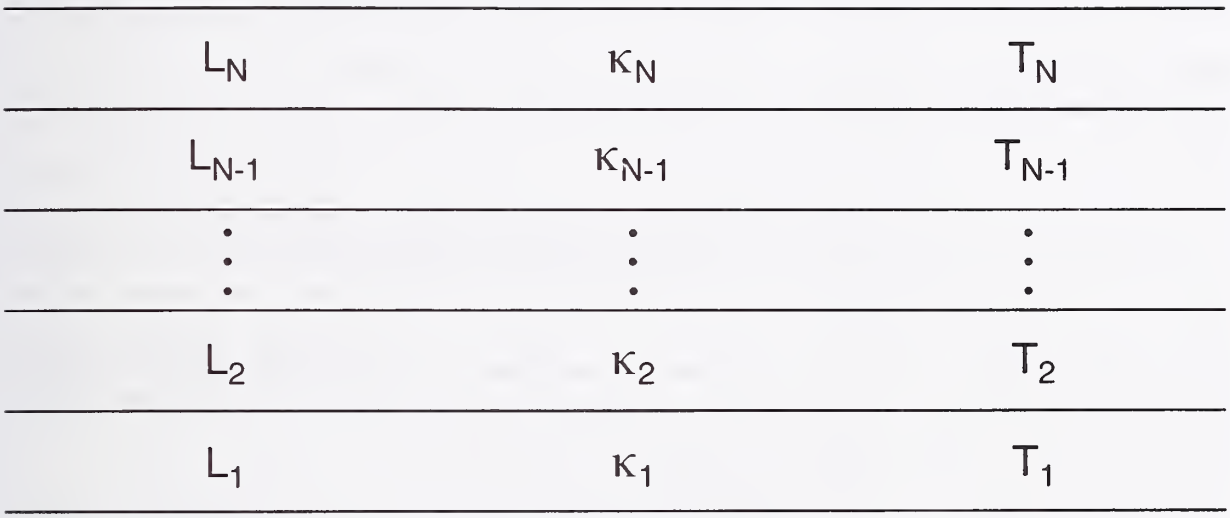

Figure 5. Schematic representation of the geometry used in multilayer analysis of the steady-state heat flow problem. In this figure, L_i, κ_i, and T_i are the thickness, thermal conductivity, and temperature, respectively, in the i-th layer.

$$A_N(\lambda) = \frac{\sigma_{N-1}\tanh(\lambda d_N) + \sigma_N A_{N-1}(\lambda)}{\sigma_{N-1} + \sigma_N A_{N-1}(\lambda)\tanh(\lambda d_N)}. \tag{121}$$

The recursion relation is most commonly used in this form. This is in part due to the hyperbolic tangent function which is bounded between (0,1), making the numerical implementation relatively easy. This coupled with the quadrature techniques introduced by Berkowitz and Lux has moved the analysis into everyday applicability.

3.3 - THE N-LAYER THERMAL PROBLEM

The depth-dependent portion of the multilayer geometry for the thermal problem is shown in figure 5. Figures 4 and 5 are presented to reinforce the connection between the electrical and thermal problems and the subsequent development of the thermal recursion relation.

From the development of Part 1, it is possible to show that the temperature on the top surface of the N-layer structure may be written as

$$T_N(x,y,0) = P_0 \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{4U(n,m)\tau_N(n,m,0)\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1)L_x L_y \kappa_N \gamma}. \tag{122}$$

The key elements in eq (122) are the Fourier coefficients, $\tau_N(\gamma)$, which depend upon the thermal conductivities and thicknesses of the layers in the multilayer structure (through the solution of the above system of simultaneous equations).

As discussed previously, the solution of eq (122) for cases beyond three layers presented a major difficulty. The analogous difficulty with the solution of the multilayer Laplace equation and its removal by means of the recursion relation provides the impetus for the introduction of a recursive scheme in the solution of multilayer thermal equation.

It is important to note that the factor of γ has been explicitly written in the denominator of eq (122). This has been done to simplify the discussion of the recursion relation described in the next section. This recursion relation will be used to take into account the multilayer thicknesses and thermal conductivities.

3.4 - THE THERMAL RECURSION RELATION

The strong resemblances of eqs (22) and (114), eqs (24) and (116), and eqs (25) and (117) lead to the possibility of expressing the Fourier expansion coefficient for the surface temperature of an N-layer structure in the form of eq (121) with the appropriate transcription from electrical conductivities to thermal conductivities, λ to γ, $A_N(\lambda)$ to $\tau_N(\gamma)$, and d_N to L_N. Then the thermal recursion relation would take the form

$$\tau_N(\gamma) = \frac{\kappa_{N-1}\tanh(\gamma L_N) + \kappa_N \tau_{N-1}(\gamma)}{\kappa_{N-1} + \kappa_N \tau_{N-1}(\gamma)\tanh(\gamma L_N)}. \tag{123}$$

As the electrical and thermal bottom-layer boundary conditions are of different form, it is necessary to evaluate the corresponding $\tau_1(\gamma)$. The form of $\tau_1(\gamma)$ may be obtained from eq (92) by setting $z = 0$ and then setting the thickness of the two top layers equal to zero. The result is that

$$\tau_1(\gamma) = \tanh(\gamma L_1). \tag{124}$$

Equations (123) and (124) are the central results of the determination of the thermal recursion relation. They provide for the determination of the Fourier coefficients of the surface temperature for an N-layer structure from the set of thicknesses and thermal conductivities $(L_i, \kappa_i, i = 1, N)$ by repeated use of the thermal recursion relation. This process begins with the one-layer Fourier coefficient as given by eq (124). This is substituted into eq (123) to generate the two-layer Fourier coefficient. The two-layer Fourier coefficient is substituted into eq (123) to determine the three-layer Fourier coefficient. This process is repeated until the Fourier coefficient for the desired number of layers is generated. Note that there is no restriction on the number of layers. This means that the exact steady-state surface temperature can now be determined for a multilayer structure with an arbitrary number of layers. It is important to note that this exact steady-state surface temperature satisfies the heat flow equation as well as the necessary boundary conditions. It does not need verification with existing layered solutions.

The recursion relation technique should complement and possibly extend some of the other efforts [25-29] aimed at problems beyond three layers.

3.5 - RELATION TO PREVIOUS SOLUTIONS

It is instructive to compare and connect the results of the thermal recursion relation with those presented in the literature. The principal one is based upon the case of a three-layer structure where the result of the thermal recursion relation is compared with that obtained from the Kokkas model and the TXYZ code. Using eqs (123) and (124), it is straightforward to show that the Fourier coefficient for the three-layer structure is given by

$$\tau_3(\gamma) =$$

$$\frac{\kappa_2\kappa_3\tanh(\gamma L_1) + \kappa_1\kappa_3\tanh(\gamma L_2) + \kappa_1\kappa_2\tanh(\gamma L_3) + \kappa_2^2\tanh(\gamma L_1)\tanh(\gamma L_2)\tanh(\gamma L_3)}{\kappa_2\kappa_3\tanh(\gamma L_1)\tanh(\gamma L_3) + \kappa_1\kappa_3\tanh(\gamma L_2)\tanh(\gamma L_3) + \kappa_2^2\tanh(\gamma L_1)\tanh(\gamma L_2) + \kappa_1\kappa_2}. \tag{125}$$

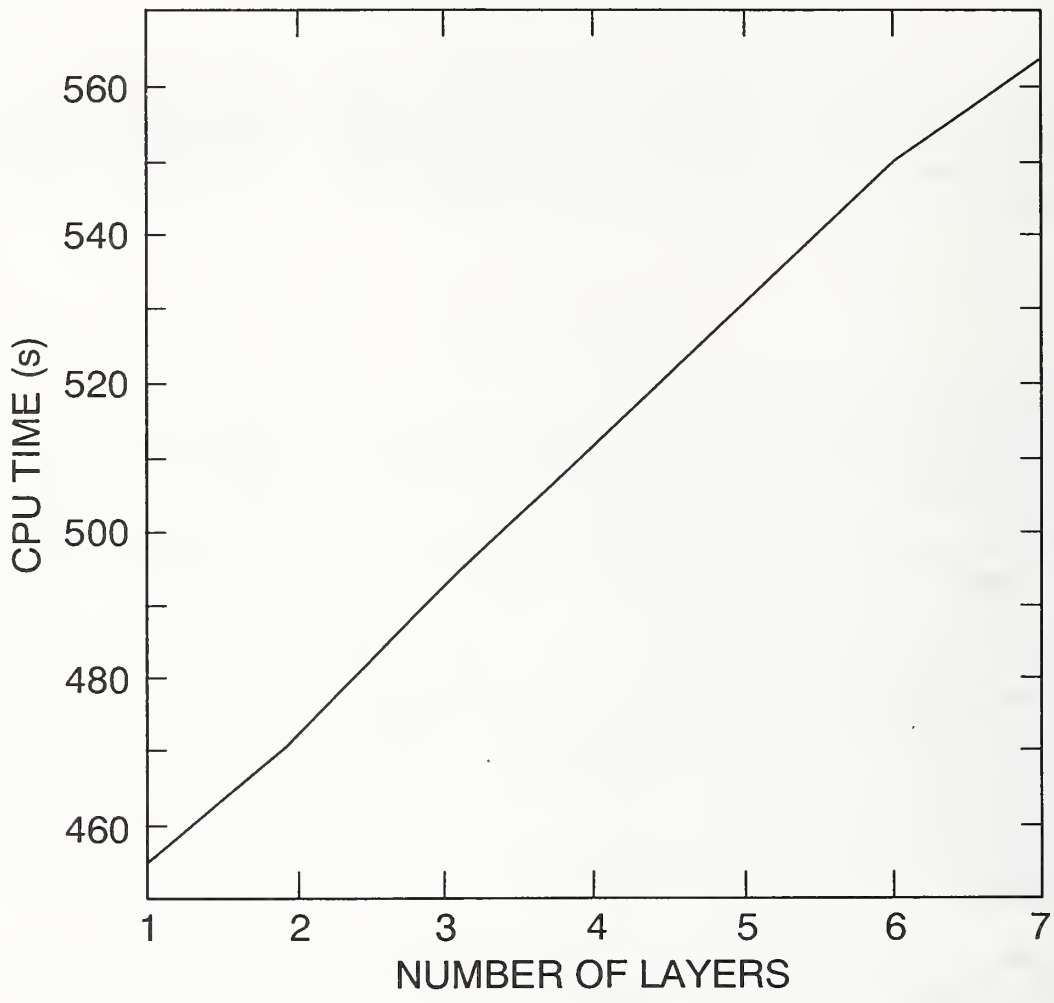

Figure 6. Example of the small increase in computation time associated with increasing the number of layers in the thermal structure. The slope represents about a 5-percent per layer increase.

It is also straightforward to show that this is in exact formal agreement with the Kokkas and TXYZ expressions for $z = 0$ for the three-layer case. This does not serve as a limited verification of the recursion relation but rather shows that the two are in agreement, as expected. In addition to this exact formal agreement, numerical simulations have been performed with the TXYZ code and a preliminary code based upon the recursion relation. As expected, these are in numerical agreement. One interesting result is that the recursion relation code actually runs about 15 percent faster. This is due to the more compact nature of the recursion relation.

It is interesting to note in eq (125) that if $\kappa_1 = \kappa_2 = \kappa_3$, then $\tau_3(\gamma)$ is independent of the thermal conductivities and reduces to $\tanh(\gamma(L_1 + L_2 + L_3))$ which is the result for a single layer of thickness equal to $L_1 + L_2 + L_3$. This line of reasoning can be extended to the N-layer equation. For the case where the thermal conductivities of the layers become equal, repeated use of the addition formula for the hyperbolic tangent shows that the form of the Fourier coefficient becomes $\tanh(\gamma(L_1 + \cdots + L_N))$ as expected for a single layer of thickness equal to $L_1 + \cdots + L_N$.

3.6 - NUMERICAL IMPLEMENTATION OF THERMAL MULTILAYER CODE, TML

The numerical implementation of the thermal recursion relation solution is contained in the Thermal MultiLayer, TML, code. The FORTRAN source code is listed in the appendix and is also contained in the HOTPAC software package.

For the sake of illustrating some of the features of the code, the TML program has been used in calculations of the surface temperature for structures of up to seven layers. The case of seven layers is used to provide a convenient range and does not represent a limitation of the recursion relation. Any number of layers is possible and can be easily achieved on modern computing systems. These temperature calculations were performed for the purpose of testing the computational efficiency of the recursive scheme. The results are presented in figure 6 and were obtained on a VAX 11/785. They are shown for the purpose of illustration of the utility of the recursion relation. There is only about a 5 percent increase in computation time for each layer added to the structure. This is strong evidence for the effectiveness of the algebraic nature of the recursion relation.

These test cases were also run for the situation where all the thermal conductivities were equal. In keeping with the above discussion, the results reduced to those for a single layer of thickness equal to $L_1 + \cdots + L_N$. This provided further support of the recursion relation and its numerical implementation.

For the purposes of acquainting the user with the TML code, a number of examples are contained in the files. The particular files, tmlio.1, tmlio.2, tmlio.3, contain the three-layer problem contained in the corresponding TXYZ30 related files, txyz30io.1, txyz30io.2, and txyz30io.3. The TML code, like the TXYZ30 code, contains the point function, line average and area average options. These are illustrated by the above.

For the purpose of showing the use of TML for real multilayer cases, the files tmlio.120,

tmlio.220, and tmlio.320 are included in the software package. These illustrate the case of the TML calculation for a 20-layer case in the point function, line average and area average modes.

3.7 - GENERALIZED ONE-DIMENSIONAL THERMAL RESISTANCE

The special case of uniform surface coverage by a single heat source for a three-layer structure has been presented in section 1.8 of this report. There it was shown that only the $\gamma \rightarrow 0 (n = 0, m = 0)$ term needs to be considered. This same line of reasoning applies to the full N-layer case. Investigation of the recursion relation for small values of γ shows for the case of uniform surface coverage by a single heat source that

$$T_N(0) = P_0 \sum_{i=1}^{N} \frac{L_i}{\kappa_i}, \tag{126}$$

which is just the generalized one-dimensional thermal resistance result.

In section 2.4, the average surface temperature (over the entire top surface ($z = 0$) area of the structure) was calculated for the case of a single heat source of dimensions lx by ly. The result is presented in eq (109) and is just the one-dimensional thermal resistance scaled by the ratio of the heat source area and the total area. Using the analysis used to obtain eq (126), it is easy to show that the average surface temperature for the N-layer structure is

$$\langle T_N(0) \rangle_a = P_0 \frac{lx \cdot ly}{L_x L_y} \sum_{i=1}^{N} \frac{L_i}{\kappa_i}, \tag{127}$$

which is just the generalized one-dimensional thermal resistance scaled by the ratio of the heat source area and the total area.

3.8 - APPLICATION TO BURIED OXIDE STRUCTURE

The thermal recursion relation should be especially useful in the understanding of the effects of multilayer structure thickness and thermal conductivities on the surface temperature. The possibility of using the surface temperature as a probe of the thermal conductivities of the structure is particularly intriguing. An example of the use of this technique is found in the case of the steady-state surface temperature of a buried oxide structure. Recent trends in semiconductor fabrication make use of the buried oxide as a way of electrically isolating the top active region. Probably the most popular version of this is found in SIMOX (Separation by IMplanted OXygen) structures where the buried oxide is formed by high-dose implantation of oxygen followed by high-temperature annealing [30]. The depth of the buried oxide is controlled by the energy of the incident oxygen ion beam.

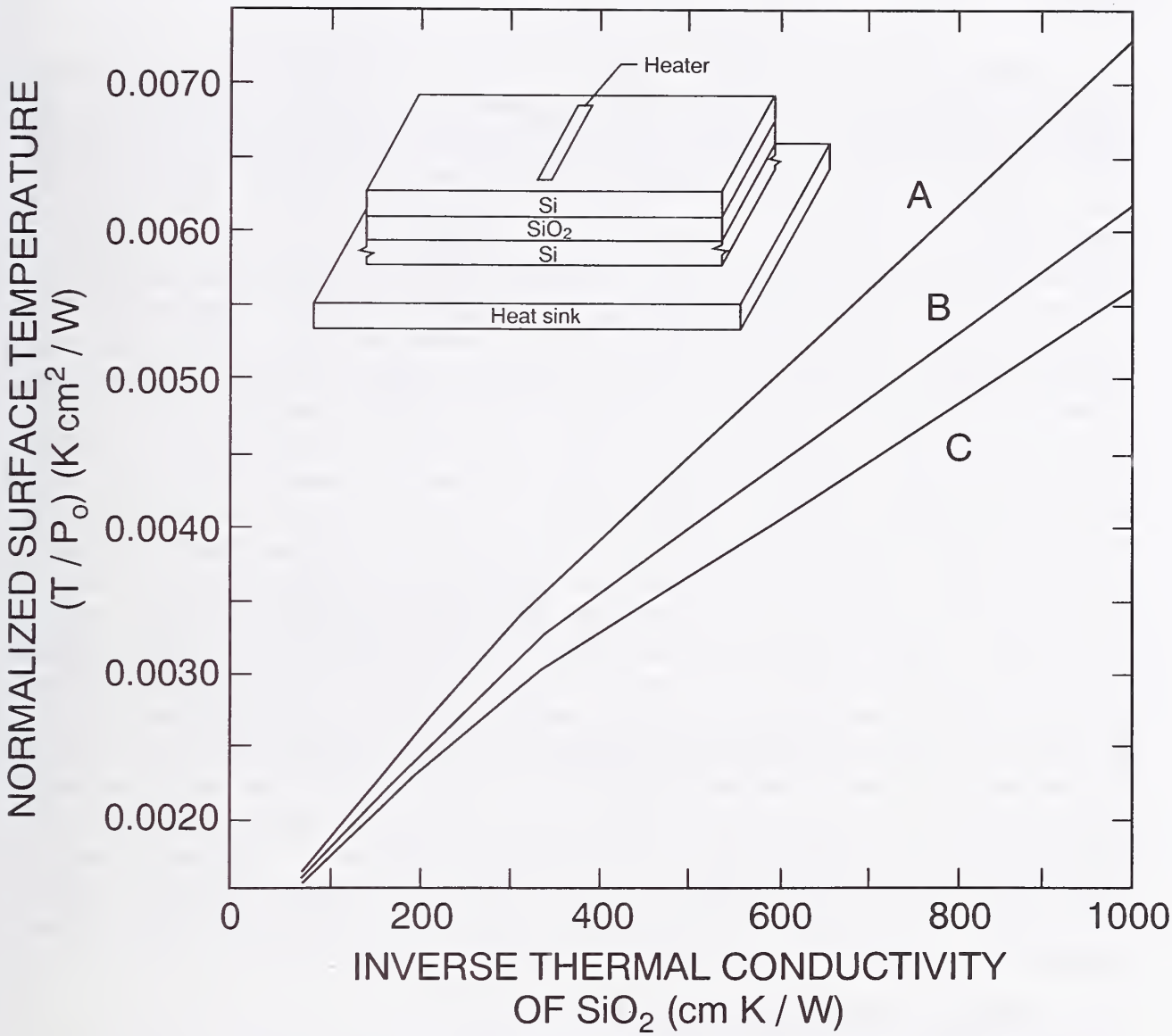

Figure 7. Example of the application of the thermal recursion relation to the calculation of the surface temperature of an SOI (Silicon-On-Insulator) structure. The insert (upper left) depicts the structure where the layer thicknesses and heater dimensions are not to scale. The buried SiO_2 is 0.00001 cm thick. Curves A, B, and C are for the cases where the surface layers of Si are 0.00001 cm, 0.00002 cm, and 0.00003 cm thick, respectively. Notice how the temperatures do not follow a one-dimensional interpretation in keeping with the two-dimensional heat flow in the more thermally conductive surface silicon layer.

When processed correctly, the implantation damage should be removed and a buried layer of SiO_2 should be formed. Electrical isolation is certainly achieved but the thermal conductivity of the buried oxide may impose a price of thermal isolation which may give rise to higher-than-desired operating temperatures in the device. These kinds of structures are prime candidates for the investigation of self-heating effects in multiple-layered structures [31,32].

As an example of the recursion relation technique, preliminary calculations were performed on a series of buried oxide structures. These were done to ascertain the possibility of using surface temperature measurements as a probe of the thermal conductivity of the buried oxide. The basic structure was modeled as a chip (rectangular structure) with lateral dimensions of 1.0275 cm by 0.7737 cm. The vertical structure consisted of a surface layer of Si over a buried layer of SiO_2 over a substrate layer of Si. The buried oxide was 0.00001 cm thick and the substrate Si layer was 0.03189 cm thick. Three thicknesses of the surface Si layer were used in the calculations. These included 0.00001 cm, 0.00002 cm, and 0.00003 cm, and are typical of SIMOX structures. The thermal conductivities of the surface and substrate Si layers were taken as the room temperature value of 1.55 W/cm°K and the thermal conductivities of the buried oxide were taken in the range from 0.001 to 0.015 W/cm°K . A small resistive heat source on the top surface was modeled as a rectangular element of lateral dimension of 0.001005 cm by 0.0825 cm. The temperature (normalized by the power density, T/P_0) was calculated in the area of the heat source for the ranges of surface Si thickness and buried oxide thermal conductivities. This was done to ascertain: 1) the effects of the surface silicon thickness and 2) the sensitivity of the calculated temperature on the buried oxide thermal conductivity. The results are presented in figure 7 where the calculated surface temperatures (normalized by P_0) in the area of the small surface heat source are plotted against the inverse thermal conductivity of the buried oxide for the three surface silicon thicknesses. These results indicate the important dependence on the thickness of the surface silicon layer. In addition, the calculated temperature decreases with increasing thickness of the surface silicon layer. If the heat flow were one-dimensional and described by eq (126), then the calculated temperature should increase with increasing thickness of the surface silicon layer. The fact that the trend is in the opposite direction points to the two-dimensional heat flow caused by the more thermally conductive surface silicon layer. This necessitates careful interpretation of the temperature data on these structures.

3.9 - SUMMARY AND CONCLUSIONS

Using the strong mathematical similarity of the multilayer Laplace equation of spreading resistance and the multilayer steady-state heat flow equation, a recursion relation has been developed for the multilayer thermal problem. There are no restrictions on the number of layers which can be used in the calculations. This makes the results truly multilayer. This recursion relation has been shown to produce the Kokkas-TXYZ results for the three-layer case. It also gives rise to a generalized one-dimensional thermal resistance result for the case of uniform surface coverage. A preliminary program based on this recursive scheme has been shown to provide very good computational speed. As an example, it has been

applied to investigate the possible determination of the buried oxide thermal conductivity in SIMOX-type structures. This recursion relation technique is simple, elegant, and powerful and should be extremely useful in the investigation and understanding of the effects of layer thicknesses and thermal conductivities on the steady-state surface temperatures of multilayer structures [33, 34].

AVAILABILITY OF HOTPAC SOFTWARE PACKAGE

The source codes for the HOTPAC software package are written in FORTRAN77 and have been run on a VAX 11/785 minicomputer system as well as a Sun SPARC10 workstation. These codes should run on other systems with little, if any, need for modification.

There are two programs contained in the HOTPAC software package. The FORTRAN77 source code and sample input and output data files are available in ASCII format on DOS-formatted floppy disks. This package is self-contained and is straightforward to run once the FORTRAN is compiled and linked by the user-supplied software. The sample input and output data files are included so that the user can check the programs for proper operation as well as to become acquainted with the setup and use of the codes.

For more information or to receive a copy of the code and the report, please contact:

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Dave Blackburn for using the code for thick middle layers which showed the need to determine a numerically stable form for the FUNZ function for this situation.

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Jim Wilson and colleagues at Texas Instruments for showing how the TXYZ code could be used as an exact benchmark for other programs.

All of the TXYZ/TXYZ20 users for making use of the code. He hopes that TXYZ/TXYZ20 users become HOTPAC (TXYZ30/TML) users.

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REFERENCES

[1] H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, Second Edition (Oxford University Press, 1959).

[2] R. V. Churchill, *Fourier Series and Boundary Value Problems*, Second Edition (McGraw-Hill, 1963).

[3] A. G. Kokkas, Analysis and Design of Electrothermal Integrated Circuits, Ph.D. Thesis, MIT, 1972.

[4] A. G. Kokkas, Thermal Analysis of Multiple-Layer Structures, *IEEE Trans. Electron Devices* **ED-21**, pp. 674-681, 1974.

[5] F. Ayres Jr., *Matrices* (Schaum Publishing Co., 1950).

[6] J. Albers, *Semiconductor Measurement Technology:* TXYZ: A Program for Semiconductor IC Thermal Analysis, NBS Special Publication 400-79, 1984.

[7] J. Albers, *Semiconductor Measurement Technology:* Version 2.0 of the TXYZ Thermal Analysis Program: **TXYZ20**, NIST Special Publication 400-89, 1992.

[8] F. F. Oettinger, Thermal Evaluation of VLSI Packages Using Test Chips – A Critical Review, *Solid State Technology* (February 1984).

[9] F. F. Oettinger and D. L. Blackburn, *Semiconductor Measurement Technology:* Thermal Resistance Measurements, NIST Special Publication 400-86 (July 1990).

[10] H. A. Schafft and J. Albers, Thermal Interactions Between Electromigration Test Structures, *IEEE Proceedings on Microelectronic Test Structures*, Vol. **1**, No. 1, pp. 132-137 (February 1988).

[11] J. L. Wright, B. W. Marks and K. D. Decker, Modeling of MMIC Devices for Determining MMIC Channel Temperatures During Life Tests, *Proceedings of the Seventh IEEE SEMI-THERM Symposium*, pp. 131-139 (1991).

[12] Certain commercial equipment, instruments, or materials are identified in this paper in order to specify the procedure adequately. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose. In spite of the author's experience that the programs performs correctly with every set of data which has been tried, there can be no assurance that the program will perform equally well on all (possibly anomalous) data. Therefore, both the author and NIST assume no liability for any possible losses resulting from the use of this program.

[13] H. A. Schafft and J. S. Suehle, The Measurement, Use and Interpretation of the

Temperature Coefficient of Resistance of Metallizations, *Solid State Electronics* **35**, No. 3, pp. 403-410 (1992).

[14] H. A. Schafft, J. S. Suehle, and P. G. Mirel, Thermal Conductivity Measurements of Thin-Film Silicon Dioxide, *Proceedings IEEE International Conference on Microelectronic Test Structures* **2**, No. 1, pp. 121-125 (March 1989).

[15] D. L. Blackburn, An Electrical Technique for the Measurement of the Peak Junction Temperature of Power Transistors, *Proceedings of the 13th IEEE International Reliability Physics Symposium*, pp. 142-150 (1975).

[16] C. Z. Lu, M. Z. Wang, X. Gui, G. B. Gao, and B. J. Yi, Fast Measurement of the Peak Junction Temperature of Power Transistors Using Electrical Method, *Proceedings of the Seventh IEEE SEMI-THERM Symposium*, pp. 39-43 (1991).

[17] M. Z. Wang, C. Z. Lu, Z. H. Cheng, Z. Wang, S. W. Feng, G. Y. Ding, X. X. Li, and G. B. Gao, Electrically Measuring the Peak Channel Temperature of Power GaAs MESFET, *Proceedings of the Ninth IEEE SEMI-THERM Symposium*, pp. 112-116 (1993).

[18] P. A. Schumann and E. E. Gardner, Application of Multilayer Potential Distribution to Spreading Resistance Correction Factors, *Journal of the Electrochemical Society* **116**, pp. 87-91, 1969.

[19] E. E. Gardner and P. A. Schumann, Spreading Resistance Correction Factors, *Solid-State Electronics* **12**, pp. 371-375, 1969.

[20] S. C. Choo, M. S. Leong, and K. L. Kuan, On the Calculation of Spreading Resistance Correction Factors, *Solid-State Electronics* **19**, pp. 561-565, 1976.

[21] O. Koefoed, *The Application of the Kernel Function in Interpretation of Geoelectrical Resistivity Measurements*, Berlin-Stuttgart, Gebrüder Borntraeger, 1968.

[22] E. D. Sunde, *Earth Conduction Effects in Transmission Systems*, New York, Dover, 1968.

[23] H. L. Berkowitz and R. A. Lux, An Efficient Integration Technique for Use in the Multilayer Analysis of Spreading Resistance Profiles, *Journal of the Electrochemical Society* **128**, pp. 1137-1141, 1981.

[24] J. Albers and H. L. Berkowitz, *Semiconductor Measurement Technology:* A Collection of Computer Programs for Two-Probe Resistance (Spreading Resistance) and Four-Probe Resistance Calculations, **RESPAC**, NIST Special Publication 400-91, 1993.

[25] G. N. Ellison, *Thermal Computations for Electronic Equipment* New York, Van Nostrand Reinhold, 1984.

[26] G. N. Ellison, TAMS - A Thermal Analyzer for Multilayer Structures, *ElectroSoft* **1**, pp. 85-97, 1990.

[27] G. N. Ellison, Extensions of a Closed Form Method for Substrate Thermal Analyzers to Include Thermal Resistances from Source-to-Substrate and Source-to-Ambient, *Proceedings of the Seventh IEEE SEMI-THERM Symposium*, pp. 140-145, 1991.

[28] C. C. Lee, J. Min, and A. L. Palisoc, An Integration Algorithm for the Temperature Solution of the Four-Layer Infinite Plate Structure, *Proceedings of the Fifth IEEE SEMI-THERM Symposium*, pp. 48-51, 1989.

[29] C. C. Lee and D. H. Chien, Thermal and Package Design of High Power Laser Diodes, *Proceedings of the Ninth IEEE SEMI-THERM Symposium*, pp. 75-80, 1993.

[30] G. K. Celler and A. E. White, Buried Oxide and Silicide Formation by High-Dose Implantation in Silicon, *Materials Research Society Bulletin*, pp. 40-46, 1992.

[31] L. T. Su, K. E. Goodson, D. A. Antoniadis, M. I. Flik, and J. E. Chung, Measurements and Modeling of Self-Heating Effects in SOI NMOSFETS, *Proceedings of the IEEE International Electron Devices Meeting*, pp. 357-360, 1992.

[32] P. R. Ganci, J-J. J. Hajjar, T. Clark, P. Humphries, J. Lapham, and D. Buss, Self-Heating in High Performance Bipolar Transistors Fabricated on SOI Substrates, *Proceedings of the IEEE International Electron Devices Meeting*, pp. 417-420, 1992.

[33] J. Albers, An Exact Solution of the Steady-State Surface Temperature for a General Multilayer Structure, *Proceedings of the Tenth IEEE SEMI-THERM Symposium*, pp. 129-137, 1994.

[34] J. Albers, An Exact Recursion Relation Solution for the Steady-State Surface Temperature of a General Multilayer Structure, *IEEE Transactions on Components, Packaging and Manufacturing Technology - Part A*, **18**, No. 1, pp. 31-38, 1995.

```
1
10 10
1 .1
.5 1.00
.4 .5
50 50
1
11  0.0  1.
1
1  5.  0.0
1
1  0.0  0.0
1   1.0
1  4.0  2.0  4.0  2.0
```

The above is the input used in the input.dat file for the example of a txyz30 point function calculation. This input is annotated below.

INPUT	DESCRIPTION OF INPUT
1	itype (=1 for point function calculation)
10 10	x and y dimensions of rectangular structure
1 0.1	thickness and thermal conductivity of top layer (3)
.5 1.00	thickness and thermal conductivity of middle layer (2)
.4 .5	thickness and thermal conductivity of bottom layer (1)
50 50	upper summation limits for n and m summations
1	iedgex (=1, then read in three values on next line)
11 0.0 1.	number of values, first point, increment
1	iedgey (=1, then read in three values on next line)
1 5. 0.0	number of values, first point, increment
1	iedgez (=1, then read in three values on next line)
1 0.0 0.0	number of values, first point, increment
1 1.0	number of sources and power density
1 4.0 2.0 4.0 2.0	weight, x, length along x, y, length along y

---

Below is the output contained in the output.dat file which is calculated and written by txyz30 using the above input.

STEADY-STATE THERMAL CALCULATION KOKKAS ANALYSIS
POINT FUNCTION EVALUATION OF T(X,Y,Z)

X	Y	Z	T(X,Y,Z)
0.00000	5.00000	0.00000	0.02264
1.00000	5.00000	0.00000	0.03674
2.00000	5.00000	0.00000	0.15234
3.00000	5.00000	0.00000	0.65010
4.00000	5.00000	0.00000	4.30401
5.00000	5.00000	0.00000	7.56167
6.00000	5.00000	0.00000	4.30401
7.00000	5.00000	0.00000	0.65010
8.00000	5.00000	0.00000	0.15234
9.00000	5.00000	0.00000	0.03674
10.00000	5.00000	0.00000	0.02264

LX= 10.00 LY= 10.00
THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
L3= 1.00000 L2= 0.50000 L1= 0.40000
K3= 0.10000 K2= 1.00000 K1= 0.50000
UPPER SUMMATION LIMITS NUP= 50 MUP= 50
NUMBER OF HEAT SOURCES= 1
POWER DENSITY= 1.000000
WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES

HEAT SOURCE	WTSOUR	XSOUR	YSOUR	LXSOUR	LYSOUR
1	1.00000	4.00000	4.00000	2.00000	2.00000

```
2
10 10
1 .1
.5 1.00
.4 .5
50 50
1
11  0.0  10.
1
1  5.  0.0
1
1  0.0  0.0
1   1.0
1  4.0  2.0  4.0  2.0
1
4.0  2.0
```

The above is the input used in the input.dat file for the example of a txyz30 line average calculation. This input is annotated below.

INPUT	DESCRIPTION OF INPUT
2	itype (=2 for line average calculation)
10 10	x and y dimensions of rectangular structure
1 .1	thickness and thermal conductivity of top layer (3)
.5 1.0	thickness and thermal conductivity of middle layer (2)
.4 .5	thickness and thermal conductivity of bottom layer (1)
50 50	upper summation limits for n and m summations
1	iedgex (=1, then read in three values on next line)
11 0.0 1.	number of values, first point, increment
1	iedgey (=1, then read in three values on next line)
1 50. 0.0	number of values, first point, increment
1	iedgez (=1, then read in three values on next line)
1 0.0 0.0	number of values, first point, increment
1 1.0	number of sources and power density
1 4.0 2.0 4.0 2.0	weight, x, length along x, y, length along y
1	nline (=1 for one line) as itype=2
4.0 2.0	x, length along x for line element

Below is the output contained in the output.dat file which is calculated and written by txyz30 using the above input.

```
 STEADY-STATE THERMAL CALCULATION KOKKAS ANALYSIS
 LINE AVERAGE EVALUATION OF TEMPERATURE
 LINE #     XLINE       LXLINE        Y            Z         AVE TEMP
   1      4.00000      2.00000      5.00000      0.00000      6.71515
 LX=   10.00    LY=   10.00
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L3=    1.00000  L2=    0.50000  L1=    0.40000
 K3=    0.10000  K2=    1.00000  K1=    0.50000
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 NUMBER OF HEAT SOURCES=    1
 POWER DENSITY=   1.000000
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR        XSOUR        YSOUR       LXSOUR       LYSOUR
      1        1.00000      4.00000      4.00000      2.00000      2.00000
 NUMBER OF LINES=  1
   1      4.00000      2.00000
```

```
3
10 10
1 .1
.5 1.00
.4 .5
50 50
1
11  0.0  10.
1
1  5.  0.0
1
1  0.0  0.0
1   1.0
1  4.0  2.0  4.0  2.0
1
4.0  2.0  4.0  2.0
```

The above is the input used in the input.dat file for the example of a txyz30 area average calculation. This input is annotated below.

INPUT	DESCRIPTION OF INPUT
3	itype (=3 for area average calculation)
10 10	x and y dimensions of rectangular structure
1 .1	thickness and thermal conductivity of top layer (3)
.5 1.00	thickness and thermal conductivity of middle layer (2)
.4 .5	thickness and thermal conductivity of bottom layer (1)
50 50	upper summation limits for n and m summations
1	iedgex (=1, then read in three values on next line)
11 0.0 1.	number of values, first point, increment
1	iedgey (=1, then read in three values on next line)
1 5. 0.0	number of values, first point, increment
1	iedgez (=1, then read in three values on next line)
1 0.0 0.0	number of values, first point, increment
1 1.0	number of sources and power density
1 4.0 2.0 4.0 2.0	weight, x, length along x, y, length along y
1	narea (=1 for one area) as itype=3
4.0 2.0 4.0 2.0	x, length along x, y, length along y for area

---

Below is the output contained in the output.dat file which is calculated and written by txyz30 using the above input.

```
 STEADY-STATE THERMAL CALCULATION KOKKAS ANALYSIS
 AREA AVERAGE EVALUATION OF TEMPERATURE
 AREA #      XAREA        LXAREA         YAREA        LYAREA       Z        AVE TEMP
   1       4.00000       2.00000       4.00000       2.00000    0.00000     5.97551
 LX=   10.00    LY=   10.00
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L3=     1.00000  L2=     0.50000  L1=     0.40000
 K3=     0.10000  K2=     1.00000  K1=     0.50000
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 NUMBER OF HEAT SOURCES=     1
 POWER DENSITY=   1.000000
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR         XSOUR         YSOUR        LXSOUR        LYSOUR
      1        1.00000       4.00000       4.00000       2.00000       2.00000
 NUMBER OF AREAS=  1
   1       4.00000       2.00000       4.00000       2.00000
```

```
1
10 10
3
1 .1
.5 1.00
.4 .5
50 50
1
11  0.0  1.
1
1  5.  0.0
1   1.0
1  4.0  2.0  4.0  2.0
```

The above is the input used in the input.dat file for the example of a tml point function calculation. This input is annotated below.

INPUT	DESCRIPTION OF INPUT
1	itype (=1 for point function calculation)
10 10	x and y dimensions of rectangular structure
3	number of layers in the structure
1 .1	thickness and thermal conductivity of layer 3
.5 1.00	thickness and thermal conductivity of layer 2
.4 .5	thickness and thermal conductivity of layer 1
50 50	upper summation limits for n and m summations
1	iedgex (=1, then read in three values on next line)
11 0.0 1.	number of values, first point, increment
1	iedgey (=1, then read in three values on next line)
1 5. 0.0	number of values, first point, increment
1 1.0	number of sources and power density
1 4.0 2.0 4.0 2.0	weight, x, length along x, y, length along y

---

Below is the output contained in the output.dat file which is calculated and written by tml using the above input.

```
 STEADY-STATE THERMAL MULTILAYER CALCULATION
 USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS
 POINT FUNCTION EVALUATION OF SURFACE T(X,Y)
      X            Y           T(X,Y)
    0.00000      5.00000      0.02264
    1.00000      5.00000      0.03674
    2.00000      5.00000      0.15234
    3.00000      5.00000      0.65010
    4.00000      5.00000      4.30401
    5.00000      5.00000      7.56167
    6.00000      5.00000      4.30400
    7.00000      5.00000      0.65010
    8.00000      5.00000      0.15234
    9.00000      5.00000      0.03674
   10.00000      5.00000      0.02264
 LX=   10.00    LY=   10.00
 NUMBER OF LAYERS IN STRUCTURE= 3
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L 3=    1.00000  K 3=    0.10000
 L 2=    0.50000  K 2=    1.00000
 L 1=    0.40000  K 1=    0.50000
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 POWER DENSITY=   1.000000
 NUMBER OF HEAT SOURCES=    1
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR        XSOUR        YSOUR       LXSOUR       LYSOUR
      1        1.00000      4.00000      4.00000      2.00000      2.00000
```

```
2
10 10
3
1 .1
.5 1.00
.4 .5
50 50
1
11  0.0  1.
1
1  5.  0.0
1   1.0
1  4.0  2.0  4.0  2.0
1
4.0  2.0
```

The above is the input used in the input.dat file for the example of a tml line average calculation. This input is annotated below.

INPUT	DESCRIPTION OF INPUT
2	itype (=2 for line average calculation)
10 10	x and y dimensions of rectangular structure
3	number of layers in the structure
1 .1	thickness and thermal conductivity of layer 3
.5 1.00	thickness and thermal conductivity of layer 2
.4 .5	thickness and thermal conductivity of layer 1
50 50	upper summation limits for n and m summations
1	iedgex (=1, then read in three values on next line)
11 0.0 1.	number of values, first point, increment
1	iedgey (=1, then read in three values on next line)
1 5. 0.0	number of values, first point, increment
1 1.0	number of sources and power density
1 4.0 2.0 4.0 2.0	weight, x, length along x, y, length along y
1	nline (=1 for one line) as itype=2
4.0 2.0	x, length along x for line element

Below is the output contained in the output.dat file which is calculated and written by tml using the above input.

```
 STEADY-STATE THERMAL MULTILAYER CALCULATION
 USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS
 LINE AVERAGE EVALUATION OF SURFACE TEMPERATURE
 LINE #      XLINE        LXLINE         Y          AVE TEMP
   1       4.00000       2.00000       5.00000       6.71515
 LX=   10.00    LY=   10.00
 NUMBER OF LAYERS IN STRUCTURE= 3
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L 3=     1.00000  K 3=     0.10000
 L 2=     0.50000  K 2=     1.00000
 L 1=     0.40000  K 1=     0.50000
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 POWER DENSITY=   1.000000
 NUMBER OF HEAT SOURCES=    1
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE     WTSOUR         XSOUR         YSOUR        LXSOUR        LYSOUR
      1         1.00000       4.00000       4.00000       2.00000       2.00000
 NUMBER OF LINES=  1
   1       4.00000       2.00000
```

```
3
10 10
3
1 .1
.5 1.00
.4 .5
50 50
1
11  0.0  1.
1
1  5.  0.0
1   1.0
1  4.0  2.0  4.0  2.0
1
4.0  2.0  4.0  2.0
```

The above is the input used in the input.dat file for the example of a tml area average calculation. This input is annotated below.

INPUT	DESCRIPTION OF INPUT
3	itype (=3 for area average calculation)
10 10	x and y dimensions of rectangular structure
3	number of layers in structure
1 .1	thickness and thermal conductivity of layer 3
.5 1.00	thickness and thermal conductivity of layer 2
.4 .5	thickness and thermal conductivity of layer 1
50 50	upper summation limits for n and m summations
1	iedgex (=1, then read in three values on next line)
11 0.0 1.	number of values, first point, increment
1	iedgey (=1, then read in three values on next line)
1 5. 0.0	number of values, first point, increment
1 1.0	number of sources and power density
1 4.0 2.0 4.0 2.0	weight, x, length along x, y, length along y
1	narea (=1 for one area) as itype=3
4.0 2.0 4.0 2.0	x, length along x, y, length along y for area

---

Below is the output contained in the output.dat file which is calculated and written by tml using the above input.

```
 STEADY-STATE THERMAL MULTILAYER CALCULATION
 USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS
 AREA AVERAGE EVALUATION OF SURFACE TEMPERATURE
 AREA #      XAREA        LXAREA         YAREA        LYAREA    AVE TEMP
   1       4.00000       2.00000       4.00000       2.00000     5.97551
 LX=   10.00    LY=   10.00
 NUMBER OF LAYERS IN STRUCTURE= 3
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L 3=    1.00000  K 3=    0.10000
 L 2=    0.50000  K 2=    1.00000
 L 1=    0.40000  K 1=    0.50000
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 POWER DENSITY=   1.000000
 NUMBER OF HEAT SOURCES=    1
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR         XSOUR         YSOUR        LXSOUR        LYSOUR
      1        1.00000       4.00000       4.00000       2.00000       2.00000
 NUMBER OF AREAS=  1
   1       4.00000       2.00000       4.00000       2.00000
```

TML I/O FILE LISTING - tmlio.120

```
1
10 10
20
     .100000    .100000
     .100000    .112202
     .100000    .125893
     .100000    .141254
     .100000    .158489
     .100000    .177828
     .100000    .199526
     .100000    .223872
     .100000    .251189
     .100000    .281838
     .100000    .316228
     .100000    .354813
     .100000    .398107
     .100000    .446683
     .100000    .501187
     .100000    .562341
     .100000    .630957
     .100000    .707945
     .100000    .794328
     .100000    .891250
50 50
1
11  0.0  1.
1
1  5.  0.0
1   1.0
1  4.0  2.0  4.0  2.0
```

The above is the input used in the input.dat file for the example of a tml point function calculation for a 20 layer structure.
This input is annotated below.

```
INPUT                    DESCRIPTION OF INPUT
1                        itype (=1 for point function calculation)
10 10                    x and y dimensions of rectangular structure
20                       number of layers in the structure
     .100000    .100000  thickness and thermal conductivity of layer 20
     .100000    .112202  thickness and thermal conductivity of layer 19
     .100000    .125893  thickness and thermal conductivity of layer 18
     .100000    .141254  thickness and thermal conductivity of layer 17
     .100000    .158489  thickness and thermal conductivity of layer 16
     .100000    .177828  thickness and thermal conductivity of layer 15
     .100000    .199526  thickness and thermal conductivity of layer 14
     .100000    .223872  thickness and thermal conductivity of layer 13
     .100000    .251189  thickness and thermal conductivity of layer 12
     .100000    .281838  thickness and thermal conductivity of layer 11
     .100000    .316228  thickness and thermal conductivity of layer 10
     .100000    .354813  thickness and thermal conductivity of layer 9
     .100000    .398107  thickness and thermal conductivity of layer 8
     .100000    .446683  thickness and thermal conductivity of layer 7
     .100000    .501187  thickness and thermal conductivity of layer 6
     .100000    .562341  thickness and thermal conductivity of layer 5
     .100000    .630957  thickness and thermal conductivity of layer 4
     .100000    .707945  thickness and thermal conductivity of layer 3
     .100000    .794328  thickness and thermal conductivity of layer 2
     .100000    .891250  thickness and thermal conductivity of layer 1
```

```
50 50                    upper summation limits for n and m summations
1                        iedgex (=1, then read in three values on next line)
11  0.0  1.              number of values, first point, increment
1                        iedgey (=1, then read in three values on next line)
1  5.  0.0               number of values, first point, increment
1   1.0                  number of sources and power density
1  4.0  2.0  4.0  2.0    weight, x, length along x, y, length along y
```

Below is the output contained in the output.dat file which is calculated and written by tml using the above input for a 20 layer structure.

```
 STEADY-STATE THERMAL MULTILAYER CALCULATION
 USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS
 POINT FUNCTION EVALUATION OF SURFACE T(X,Y)
      X            Y           T(X,Y)
    0.00000      5.00000      0.01703
    1.00000      5.00000      0.02522
    2.00000      5.00000      0.10517
    3.00000      5.00000      0.42075
    4.00000      5.00000      3.24066
    5.00000      5.00000      5.81674
    6.00000      5.00000      3.24066
    7.00000      5.00000      0.42075
    8.00000      5.00000      0.10517
    9.00000      5.00000      0.02522
   10.00000      5.00000      0.01703
 LX=   10.00    LY=   10.00
 NUMBER OF LAYERS IN STRUCTURE=20
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L20=    0.10000  K20=    0.10000
 L19=    0.10000  K19=    0.11220
 L18=    0.10000  K18=    0.12589
 L17=    0.10000  K17=    0.14125
 L16=    0.10000  K16=    0.15849
 L15=    0.10000  K15=    0.17783
 L14=    0.10000  K14=    0.19953
 L13=    0.10000  K13=    0.22387
 L12=    0.10000  K12=    0.25119
 L11=    0.10000  K11=    0.28184
 L10=    0.10000  K10=    0.31623
 L 9=    0.10000  K 9=    0.35481
 L 8=    0.10000  K 8=    0.39811
 L 7=    0.10000  K 7=    0.44668
 L 6=    0.10000  K 6=    0.50119
 L 5=    0.10000  K 5=    0.56234
 L 4=    0.10000  K 4=    0.63096
 L 3=    0.10000  K 3=    0.70794
 L 2=    0.10000  K 2=    0.79433
 L 1=    0.10000  K 1=    0.89125
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 POWER DENSITY=   1.000000
 NUMBER OF HEAT SOURCES=    1
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR        XSOUR        YSOUR       LXSOUR       LYSOUR
      1        1.00000      4.00000      4.00000      2.00000      2.00000
```

```
2
10 10
20
     .100000    .100000
     .100000    .112202
     .100000    .125893
     .100000    .141254
     .100000    .158489
     .100000    .177828
     .100000    .199526
     .100000    .223872
     .100000    .251189
     .100000    .281838
     .100000    .316228
     .100000    .354813
     .100000    .398107
     .100000    .446683
     .100000    .501187
     .100000    .562341
     .100000    .630957
     .100000    .707945
     .100000    .794328
     .100000    .891250
50 50
1
11  0.0  1.
1
1  5.  0.0
1   1.0
1  4.0  2.0  4.0  2.0
1
4.0  2.0
```

The above is the input used in the input.dat file for the example of a tml line average calculation for a 20 layer structure.
This input is annotated below.

```
INPUT                    DESCRIPTION OF INPUT
2                        itype (=2 for the line average calculation)
10 10                    x and y dimensions of rectangular structure
20                       number of layers in the structure
     .100000    .100000  thickness and thermal conductivity of layer 20
     .100000    .112202  thickness and thermal conductivity of layer 19
     .100000    .125893  thickness and thermal conductivity of layer 18
     .100000    .141254  thickness and thermal conductivity of layer 17
     .100000    .158489  thickness and thermal conductivity of layer 16
     .100000    .177828  thickness and thermal conductivity of layer 15
     .100000    .199526  thickness and thermal conductivity of layer 14
     .100000    .223872  thickness and thermal conductivity of layer 13
     .100000    .251189  thickness and thermal conductivity of layer 12
     .100000    .281838  thickness and thermal conductivity of layer 11
     .100000    .316228  thickness and thermal conductivity of layer 10
     .100000    .354813  thickness and thermal conductivity of layer 9
     .100000    .398107  thickness and thermal conductivity of layer 8
     .100000    .446683  thickness and thermal conductivity of layer 7
     .100000    .501187  thickness and thermal conductivity of layer 6
     .100000    .562341  thickness and thermal conductivity of layer 5
     .100000    .630957  thickness and thermal conductivity of layer 4
     .100000    .707945  thickness and thermal conductivity of layer 3
     .100000    .794328  thickness and thermal conductivity of layer 2
     .100000    .891250  thickness and thermal conductivity of layer 1
```

```
50 50                    upper summation limits for n and m summations
1                        iedgex (=1, then read in three values on next line)
11  0.0  1.              number of values, first point, increment
1                        iedgey (=1, then read in three values on next line)
1  5.  0.0               number of values, first point, increment
1   1.0                  number of sources and power density
1  4.0  2.0  4.0  2.0    weight, x, length along x, y, length along y
1                        nline as itype=2 (=1 for one line, etc)
4.0  2.0                 x, length along x for line element
```

Below is the output contained in the output.dat file which is calculated and written by tml using the above input for a 20 layer structure.

```
 STEADY-STATE THERMAL MULTILAYER CALCULATION
 USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS
 LINE AVERAGE EVALUATION OF SURFACE TEMPERATURE
 LINE #     XLINE       LXLINE        Y         AVE TEMP
   1      4.00000      2.00000      5.00000      5.19144
 LX=   10.00    LY=   10.00
 NUMBER OF LAYERS IN STRUCTURE=20
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L20=    0.10000  K20=    0.10000
 L19=    0.10000  K19=    0.11220
 L18=    0.10000  K18=    0.12589
 L17=    0.10000  K17=    0.14125
 L16=    0.10000  K16=    0.15849
 L15=    0.10000  K15=    0.17783
 L14=    0.10000  K14=    0.19953
 L13=    0.10000  K13=    0.22387
 L12=    0.10000  K12=    0.25119
 L11=    0.10000  K11=    0.28184
 L10=    0.10000  K10=    0.31623
 L 9=    0.10000  K 9=    0.35481
 L 8=    0.10000  K 8=    0.39811
 L 7=    0.10000  K 7=    0.44668
 L 6=    0.10000  K 6=    0.50119
 L 5=    0.10000  K 5=    0.56234
 L 4=    0.10000  K 4=    0.63096
 L 3=    0.10000  K 3=    0.70794
 L 2=    0.10000  K 2=    0.79433
 L 1=    0.10000  K 1=    0.89125
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 POWER DENSITY=   1.000000
 NUMBER OF HEAT SOURCES=    1
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR        XSOUR        YSOUR       LXSOUR       LYSOUR
     1        1.00000      4.00000      4.00000      2.00000      2.00000
 NUMBER OF LINES=  1
   1      4.00000      2.00000
```

```
3
10 10
20
     .100000    .100000
     .100000    .112202
     .100000    .125893
     .100000    .141254
     .100000    .158489
     .100000    .177828
     .100000    .199526
     .100000    .223872
     .100000    .251189
     .100000    .281838
     .100000    .316228
     .100000    .354813
     .100000    .398107
     .100000    .446683
     .100000    .501187
     .100000    .562341
     .100000    .630957
     .100000    .707945
     .100000    .794328
     .100000    .891250
50 50
1
11  0.0  1.
1
1  5.  0.0
1   1.0
1  4.0  2.0  4.0  2.0
1
4.0  2.0  4.0   2.0
```

The above is the input used in the input.dat file for the example of a tml area average calculation for a 20 layer structure.
This input is annotated below.

```
INPUT                    DESCRIPTION OF INPUT
3                        itype (=3 for the line average calculation)
10 10                    x and y dimensions of rectangular structure
20                       number of layers in the structure
     .100000    .100000  thickness and thermal conductivity of layer 20
     .100000    .112202  thickness and thermal conductivity of layer 19
     .100000    .125893  thickness and thermal conductivity of layer 18
     .100000    .141254  thickness and thermal conductivity of layer 17
     .100000    .158489  thickness and thermal conductivity of layer 16
     .100000    .177828  thickness and thermal conductivity of layer 15
     .100000    .199526  thickness and thermal conductivity of layer 14
     .100000    .223872  thickness and thermal conductivity of layer 13
     .100000    .251189  thickness and thermal conductivity of layer 12
     .100000    .281838  thickness and thermal conductivity of layer 11
     .100000    .316228  thickness and thermal conductivity of layer 10
     .100000    .354813  thickness and thermal conductivity of layer 9
     .100000    .398107  thickness and thermal conductivity of layer 8
     .100000    .446683  thickness and thermal conductivity of layer 7
     .100000    .501187  thickness and thermal conductivity of layer 6
     .100000    .562341  thickness and thermal conductivity of layer 5
     .100000    .630957  thickness and thermal conductivity of layer 4
     .100000    .707945  thickness and thermal conductivity of layer 3
     .100000    .794328  thickness and thermal conductivity of layer 2
     .100000    .891250  thickness and thermal conductivity of layer 1
```

```
50 50                    upper summation limits for n and m summations
1                        iedgex (=1, then read in three values on next line)
11  0.0  1.              number of values, first point, increment
1                        iedgey (=1, then read in three values on next line)
1  5.  0.0               number of values, first point, increment
1   1.0                  number of sources and power density
1  4.0  2.0  4.0  2.0    weight, x, length along x, y, length along y
1                        narea as itype=3 (=1 for one area, etc)
4.0  2.0  4.0  2.0       x, length along x, y, length along y for area element
```

Below is the output contained in the output.dat file which is calculated and written by tml using the above input for a 20 layer structure.

```
 STEADY-STATE THERMAL MULTILAYER CALCULATION
 USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS
 AREA AVERAGE EVALUATION OF SURFACE TEMPERATURE
 AREA #      XAREA        LXAREA         YAREA        LYAREA    AVE TEMP
   1       4.00000       2.00000       4.00000       2.00000     4.64315
 LX=   10.00    LY=   10.00
 NUMBER OF LAYERS IN STRUCTURE=20
 THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS
 L20=     0.10000  K20=     0.10000
 L19=     0.10000  K19=     0.11220
 L18=     0.10000  K18=     0.12589
 L17=     0.10000  K17=     0.14125
 L16=     0.10000  K16=     0.15849
 L15=     0.10000  K15=     0.17783
 L14=     0.10000  K14=     0.19953
 L13=     0.10000  K13=     0.22387
 L12=     0.10000  K12=     0.25119
 L11=     0.10000  K11=     0.28184
 L10=     0.10000  K10=     0.31623
 L 9=     0.10000  K 9=     0.35481
 L 8=     0.10000  K 8=     0.39811
 L 7=     0.10000  K 7=     0.44668
 L 6=     0.10000  K 6=     0.50119
 L 5=     0.10000  K 5=     0.56234
 L 4=     0.10000  K 4=     0.63096
 L 3=     0.10000  K 3=     0.70794
 L 2=     0.10000  K 2=     0.79433
 L 1=     0.10000  K 1=     0.89125
 UPPER SUMMATION LIMITS    NUP=   50  MUP=   50
 POWER DENSITY=   1.000000
 NUMBER OF HEAT SOURCES=    1
 WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES
 HEAT SOURCE    WTSOUR        XSOUR         YSOUR        LXSOUR        LYSOUR
      1        1.00000       4.00000       4.00000       2.00000       2.00000
 NUMBER OF AREAS=  1
   1       4.00000       2.00000       4.00000       2.00000
```

APPENDIX A - TXYZ30 LISTING

```
C*************************************************************************
C     TXYZ30 - TXYZ VERSION 3.0  - VERSION DATE 06/27/95
C
C     THIS IS VERSION 3.0 OF THE TXYZ THERMAL ANALYSIS CODE.  THIS VERSION
C     RETAINS THE CHANGES MADE IN VERSION 2.0 (ASSIGNMENT OF ARBITRARY WEIGHTS,
C     BOTH + AND -, TO THE HEAT SOURCES ON THE SURFACE AND THE FIX OF A PROBLEM
C     WITH THICK MIDDLE LAYERS) AND INCORPORATES THE FOLLOWING ADDITIONS:
C     1) ANALYTIC AVERAGING OF THE TEMPERATURE OVER ARBITRARY LINE SEGMENT(S),
C     2) ANALYTIC AVERAGING OF THE TEMPERATURE OVER ARBITRARY AREA(S),
C     3) RENUMBERING OF LAYERS TO BE IN KEEPING WITH THE NOTATION TO BE USED
C        IN THE MULTILAYER THERMAL PROBLEM (SEE TML CODE).
C     THE LINE AND AREA AVERAGING ARE PERFORMED ANALYTICALLY FOR UNIFORM
C     WEIGHTING OF THE AVERAGE.  THE AREA AVERAGE SHOULD PROVIDE A CLOSER
C     LINK WITH EXPERIMENT WHERE THE MEASUREMENT DOES AN AVERAGING OVER
C     SOME AREA ELEMENT (ELECTRICALLY OR OPTICALLY).
C
C     THIS PROGRAM CALCULATES THE STEADY-STATE TEMPERATURE DISTRIBUTION
C     FOR A RECTANGULAR THREE LAYER STRUCTURE WITH AN ARBITRARY NUMBER
C     OF RECTANGULAR HEAT SOURCES/SINKS ON THE TOP SURFACE.
C     THE CALCULATION FOLLOWS FROM THE INPUT OF THE THICKNESSES AND THERMAL
C     CONDUCTIVITIES OF THE THREE LAYERS.
C     THE TEMPERATURE (RELATIVE TO THAT OF THE BOTTOM HEAT SINK) MAY BE
C     CALCULATED AS A POINT FUNCTION, A LINE AVERAGE, OR AN AREA AVERAGE
C     ANYWHERE ON OR INSIDE THE THREE-LAYER STRUCTURE.
C     IT IS IMPORTANT TO EMPHASIZE THAT THE CALCULATION IS GENERAL FOR THE
C     THREE LAYER STRUCTURE AND THE APPLICATION TO SEMICONDUCTOR STRUCTURES
C     IS A SPECIAL CASE.
C     THE STARTING EQUATIONS USED ARE GIVEN IN EQUATIONS (13)-(23), WITH S=0
C     (ZERO FREQUENCY, STEADY-STATE CONDITION), IN THE PAPER BY KOKKAS (BELOW).
C     THE LINE AND AREA AVERAGES FOLLOW FROM ANALYTICAL EVALUATION OF THESE
C     EQUATIONS.
C
C     REFERENCES: THE ORIGINAL MATHEMATICAL ANALYSIS OF THE THREE-LAYER
C                 STRUCTURE WAS PERFORMED IN THE PAPER "THERMAL ANALYSIS
C                 OF MULTIPLE-LAYERED STRUCTURES" BY ACHILLES G. KOKKAS,
C                 IEEE TRANS. ELEC. DEV. VOL. ED-21, NO. 11, 674-681 (1974).
C                 THIS PAPER WAS DRAWN FROM HIS PHD THESIS: A. G. KOKKAS,
C                 "ANALYSIS AND DESIGN OF ELECTROTHERMAL INTEGRATED CIRCUITS,"
C                 PH.D. THESIS, MIT, 1972.
C
C                 THE ORIGINAL FORTRAN IMPLEMENTATION OF THE STEADY STATE
C                 KOKKAS EQUATIONS IS CONTAINED IN THE TXYZ CODE AND
C                 WAS PRESENTED IN THE REPORT "SEMICONDUCTOR MEASUREMENT
C                 TECHNOLOGY: TXYZ: A PROGRAM FOR SEMICONDUCTOR IC THERMAL
C                 ANALYSIS"  BY JOHN ALBERS, NBS SPECIAL PUBLICATION 400-76
C                 (APRIL 1984).
C
C                 VERSION 2.0 OF THE TXYZ CODE WAS PRESENTED IN THE REPORT
C                 "SEMICONDUCTOR MEASUREMENT TECHNOLOGY: VERSION 2.0 OF THE
C                 TXYZ THERMAL ANALYSIS PROGRAM: TXYZ20" BY JOHN ALBERS,
C                 NIST SPECIAL PUBLICATION 400-89 (JUNE 1992).
C
C                 VERSION 3.0 OF THE TXYZ CODE AND THE ACCOMPANYING THERMAL
C                 MULTILAYER CODES ARE DISCUSSED IN THIS REPORT
C                 "SEMICONDUCTOR MEASUREMENT TECHNOLOGY: HOTPAC: PROGRAMS
C                 FOR THERMAL ANALYSIS INCLUDING VERSION 3.0 OF THE TXYZ
C                 PROGRAM, TXYZ30, AND THE THERMAL MULTILAYER PROGRAM, TML"
C                 BY JOHN ALBERS, NIST SPECIAL PUBLICATION 400-96.
C*************************************************************************
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C     IN THE PRESENT FORM, THE PROGRAM ALLOWS UP TO 500 TERMS TO BE INCLUDED
C     IN BOTH THE N SUM (ALONG X) AND THE M SUM (ALONG Y).  TO GO BEYOND
C     THIS NUMBER, SUBSTITUTE THE FOLLOWING TWO LINES WITH THE APPROPRIATE
C     VALUES OF NX AND MY FOR THE FIRST TWO DIMENSION STATEMENTS - MAKE SURE TO
C     REMOVE THE COMMENTS FROM THE NEW LINES AND COMMENT OUT THE REPLACED LINES.
C      DIMENSION X(100), Y(100), Z(100), COSYT(MY)
C      DIMENSION ARUZER(NX,MY), ARFUNZ(NX,MY)
C     ALSO REPLACE THE TESTING LINE ABOVE THE LINE LABELLED 110 WITH
C      IF (NUP.GT.NX.OR.MUP.GT.MY) GO TO 3999
C*******************************************************************************
      DIMENSION X(100), Y(100), Z(100), COSYT(500)
      DIMENSION ARUZER(500,500), ARFUNZ(500,500)
      DIMENSION WTSOUR(50), XSOUR(50), YSOUR(50)
      DIMENSION XLINE(30), XAREA(30), YAREA(30)
      REAL LXLINE(30), LXAREA(30), LYAREA(30)
      REAL LXSOUR(50), LYSOUR(50), K3, K2, K1, LX, LY, L3, L2, L1
      COMMON  K3, K2, K1, LX, LY, L3, L2, L1
      COMMON NSOUR,WTSOUR,XSOUR,YSOUR, LXSOUR, LYSOUR
C     INPUT DATA IS READ FROM 10 AND OUTPUT GOES TO 12.
C     THESE ARE WRITTEN IN LOWER CASE.  MANY OPERATING SYSTEMS TAKE
C     UPPER AND LOWER CASE AS EQUIVALENT.  THE UNIX OPERATING SYSTEM
C     VIEWS THE UPPER AND LOWER CASE NAMED FILES AS DIFFERENT.  UNIX
C     DEFAULTS TO THE LOWER CASE WHICH IS USED HERE.
      open( unit=10, file='input.dat', status = 'unknown')
      open( unit=12, file='output.dat', status = 'unknown')
      PI=3.14159265
 2    FORMAT(1X,'STEADY-STATE THERMAL CALCULATION KOKKAS ANALYSIS')
 3    FORMAT(1X,'THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS')
 4    FORMAT(1X,'K3= ',F10.5,'  K2= ',F10.5,'  K1= ',F10.5)
 5    FORMAT(1X,'L3= ',F10.5,'  L2= ',F10.5,'  L1= ',F10.5)
 6    FORMAT(1X,'UPPER SUMMATION LIMITS ',2X,' NUP=',I5,
     1 '  MUP=',I5)
 7    FORMAT(1X,'NUMBER OF HEAT SOURCES=',I5)
 8    FORMAT(1X,'WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES')
 9    FORMAT(1X,'HEAT SOURCE',4X,'WTSOUR',8X,'XSOUR',8X,'YSOUR',7X,
     1 'LXSOUR',7X,'LYSOUR')
10    FORMAT(4X,I3,5X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
11    FORMAT(1X,'POWER DENSITY=',F11.6)
27    FORMAT(1X,'LX= ',F7.2,3X,' LY= ',F7.2)
22    FORMAT(1X,F12.4,2X,F12.4,2X,F12.4,2X,F12.4)
31    FORMAT(1X,6I7)
32    FORMAT(1X,'NUMBER OF LINES=',I3)
33    FORMAT(1X,I3,3X,F10.5,3X,F10.5)
34    FORMAT(1X,'NUMBER OF AREAS=',I3)
35    FORMAT(1X,I3,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
40    FORMAT(1X,'POINT FUNCTION EVALUATION OF T(X,Y,Z)')
41    FORMAT(1X,'LINE AVERAGE EVALUATION OF TEMPERATURE')
42    FORMAT(1X,'AREA AVERAGE EVALUATION OF TEMPERATURE')
43    FORMAT(6X,'X',12X,'Y',12X,'Z',9X,'T(X,Y,Z)')
44    FORMAT(1X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
45    FORMAT(1X,'LINE #',5X,'XLINE',7X,'LXLINE',8X,'Y',12X,'Z',9X,
     1'AVE TEMP')
46    FORMAT(1X,I3,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5,3x,f10.5)
47    FORMAT(1X,'AREA #',5X,'XAREA',7X,'LXAREA',8X,'YAREA',7X,
     1 'LYAREA',6X,'Z',7x,'AVE TEMP')
48    FORMAT(1X,I3,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5,1X,F10.5,
     11X,F10.5)
```

```
 51   FORMAT(1X,I4,3X,I4)
 52   FORMAT(1X,F10.5,3X,F10.5)
 53   FORMAT(1X,I1)
 54   FORMAT(1X,I4,3X,F10.5,3X,F10.5)
 55   FORMAT(1X,F10.5)
 56   FORMAT(1X,I2,3X,F10.5)
 57   FORMAT(1X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
 88   FORMAT(1X,'YOUR UPPER LIMIT OF SUMMATION IS TOO LARGE. TRY AGAIN')
C*******************************************************************************
C
C                     -----INPUT SECTION-----
C     THE FOLLOWING VARIABLES ARE READ BY THE PROGRAM FROM FOR010
C
C         ITYPE-----TYPE OF ANALYSIS
C                   =1 FOR POINT FUNCTION, T(X,Y,Z), EVALUATION
C                   =2 FOR LINE AVERAGE, <T(Y,Z)>, EVALUATION
C                   =3 FOR AREA AVERAGE, <T(Z)>, EVALUATION
C         LX--------X DIMENSION OF 3 LAYER STRUCTURE
C         LY--------Y DIMENSION OF 3 LAYER STRUCTURE
C         L3--------THICKNESS OF TOP LAYER
C         K3--------THERMAL CONDUCTIVITY OF TOP LAYER
C         L2--------THICKNESS OF MIDDLE LAYER
C         K2--------THERMAL CONDUCTIVITY OF MIDDLE LAYER
C         L1--------THICKNESS OF BOTTOM LAYER
C         K1--------THERMAL CONDUCTIVITY OF BOTTOM LAYER
C         NUP-------UPPER LIMIT OF N SUM, X DIRECTION
C         MUP-------UPPER LIMIT OF M SUM, Y DIRECTION
C
C         IEDGEX----INDEX FOR GENERATING THE VALUES OF X TO BE USED
C                    =1 TO READ DATA FOR FIXED INCREMENT X VALUES
C                    =2 TO READ IN ARRAY OF X VALUES OF NONFIXED INCREMENT)
C                   IF IEDGEX=1 THEN READ THE THREE VARIABLES (ON SAME LINE)
C                       ILX  THE NUMBER OF X VALUES TO BE USED
C                       X1   THE VALUE OF THE FIRST POINT IN X
C                       STEPX (THE INCREMENT IN X)
C                   IF IEDGEX=2 THEN READ THE VARIABLE AND ARRAY (ONE PER LINE)
C                       ILX  THE NUMBER OF X VALUES TO BE USED
C                       X(I) THE ARRAY OF X VALUES (I=1,ILX)
C
C         IEDGEY----INDEX FOR GENERATING THE VALUES OF Y TO BE USED
C                    =1 TO READ DATA FOR FIXED INCREMENT Y VALUES
C                    =2 TO READ IN ARRAY OF X VALUES OF NONFIXED INCREMENT)
C                   IF IEDGEY=1 THEN READ THE THREE VARIABLES (ON SAME LINE)
C                       ILY  THE NUMBER OF Y VALUES TO BE USED
C                       Y1   THE VALUE OF THE FIRST POINT IN Y
C                       STEPY (THE INCREMENT IN Y)
C                   IF IEDGEY=2 THEN READ THE VARIABLE AND ARRAY (ONE PER LINE)
C                       ILY  THE NUMBER OF Y VALUES TO BE USED
C                       Y(I) THE ARRAY OF Y VALUES (I=1,ILY)
C
```

```
C          IEDGEZ----INDEX FOR GENERATING THE VALUES OF Z TO BE USED
C                     =1 TO READ DATA FOR FIXED INCREMENT Z VALUES
C                     =2 TO READ IN ARRAY OF Z VALUES OF NONFIXED INCREMENT)
C                    IF IEDGEZ=1 THEN READ THE THREE VARIABLES (ON SAME LINE)
C                        ILZ  THE NUMBER OF Z VALUES TO BE USED
C                        Z1   THE VALUE OF THE FIRST POINT IN Z
C                        STEPZ (THE INCREMENT IN Z)
C                    IF IEDGEZ=2 THEN READ THE VARIABLE AND ARRAY (ONE PER LINE)
C                        ILZ  THE NUMBER OF Z VALUES TO BE USED
C                        Z(I) THE ARRAY OF Z VALUES (I=1,ILZ)
C
C         NOTE: ENTER THE Z-RELATED VARIABLES (Z1, STEPZ OR THE Z(I) ARRAY)
C               AS ZERO OR POSITIVE QUANTITIES.  THE PROGRAM CONVERTS THE FINAL
C               Z(I) ARRAY TO ZERO OR NEGATIVE QUANTITIES AS THE CALCULATION
C               TAKES THE Z VARIABLE TO BE ZERO OR NEGATIVE.
C
C         IMPORTANT: THE POINT FUNCTION EVALUATION OF THE TEMPERATURE IS
C                    THE MOST ELEMENTAL VERSION.  IN ORDER TO SIMPLIFY THE
C                    CONSTRUCTION OF THE DATA FILES FOR LINE AND AREA
C                    AVERAGES, THE PROGRAM EXPECTS TO SEE THE ABOVE IEDGEX,
C                    IEDGEY AND IEDGEZ DATA.  THIS IS READ EVEN IF IT IS NOT
C                    USED FOR THE AVERAGE VERSIONS.  HOWEVER, THE LINE AND
C                    AREA INFORMATION MAY THEN BE SIMPLY APPENDED TO THE END
C                    OF THE DATA FILE IN ORDER TO RUN THESE VERSIONS.  SEE THE
C                    SAMPLE I/O FILES FOR AN EXAMPLE OF THIS.
C
C          NSOUR-----NUMBER OF HEAT SOURCES (UP TO 50)
C          P0--------POWER DENSITY
C
C          THE NEXT NSOUR LINES READ THE FOLLOWING INFORMATION FOR THE
C          HEAT SOURCES (WITH ALL THE INFORMATION FOR EACH OF THE ELEMENTS
C                        ON A SINGLE LINE)
C
C          WTSOUR(I)-WEIGHTING FACTOR OF I-TH SOURCE
C                    (POSITIVE FOR SOURCE, NEGATIVE FOR SINK)
C          XSOUR(I)--X COORDINATE OF ORIGIN OF I-TH SOURCE
C          LXSOUR(I)-LENGTH ALONG X DIRECTION OF I-TH SOURCE
C          YSOUR(I)--Y COORDINATE OF ORIGIN OF I-TH SOURCE
C          LYSOUR(I)-LENGTH ALONG Y DIRECTION OF I-TH SOURCE
C
C          IF ITYPE=1, THE POINT FUNCTION CALCULATION CONTINUES WITH THE ABOVE
C                      SET OF (X,Y,Z) VALUES
C          IF ITYPE=2, THE LINE AVERAGE CALCULATION READS THE FOLLOWING:
C                      NLINE---THE NUMBER OF LINE SEGMENTS TO DO THE AVERAGE
C                      THE NEXT NLINE LINES THEN READ
C                      XLINE(J), LXLINE(J)--THE LOCATION AND LENGTH OF THE
C                                           J-TH LINE ELEMENT
C          IF ITYPE=3, THE AREA AVERAGE CALCULATION READS THE FOLLOWING:
C                      NAREA---THE NUMBER OF AREA SEGMENTS TO DO THE AVERAGE
C                      THE NEXT NAREA LINES THEN READ
C                      XAREA(J),LXAREA(J),YAREA(J),LYAREA(J)-THE LOCATION
C                      AND LENGTHS OF THE J-TH AREA ELMENT
C*************************************************************************
```

```
C     READ ITYPE (ANALYSIS TYPE, 1 FOR POINT, 2 FOR LINES, 3 FOR AREAS)
      READ(10,*)ITYPE
C     READ LX AND LY (THE X AND Y DIMENSIONS OF THE RECTANGULAR STRUCTURE)
      READ(10,*) LX, LY
C     READ L3 AND K3 (THICKNESS AND THERMAL CONDUCTIVITY OF THE TOP LAYER)
      READ(10,*) L3, K3
C     READ L2 AND K2 (THICKNESS AND THERMAL CONDUCTIVITY OF THE MIDDLE LAYER)
      READ(10,*) L2, K2
C     READ L1 AND K1 (THICKNESS AND THERMAL CONDUCTIVITY OF THE BOTTOM LAYER)
      READ(10,*) L1, K1
C     READ NUP AND MUP (UPPER LIMIT OF THE SUMMATION OVER THE INDEX N (X-DIR)
C                       UPPER LIMIT OF THE SUMMATION OVER THE INDEX M (Y-DIR))
      READ(10,*)NUP,MUP
C     NUP AND MUP MUST BE LESS THAN OR EQUAL TO THE DIMENSIONALITY OF
C     ARUZER AND ARFUNZ
C     THE NEXT LINE ALLOWS VALUES OF NUP AND MUP UP TO THE PRESENT DIMENSION
C     OF ARUZER AND ARFUNZ.  TO GO BEYOND THIS VALUE, THE USER SHOULD EDIT
C     THE DIMENSION STATEMENTS ACCORDINGLY AND THEN COMMENT OUT OR MODIFY
C     THE NEXT LINE OF CODE
      IF (NUP.GT.500.OR.MUP.GT.500) GO TO 3999
      READ(10,*)IEDGEX
      GOTO (110,115)IEDGEX
 110  READ (10,*)ILX,X1,STEPX
      DO 111 I=1,ILX
      X(I)=X1+(I-1)*STEPX
 111  CONTINUE
      GOTO 119
 115  READ (10,*)ILX
      DO 116 I=1,ILX
      READ(10,*)X(I)
 116  CONTINUE
 119  READ(10,*)IEDGEY
      GOTO (120,125) IEDGEY
 120  READ (10,*)ILY,Y1,STEPY
      DO 121 I=1,ILY
      Y(I)=Y1+(I-1)*STEPY
 121  CONTINUE
      GOTO 129
 125  READ (10,*)ILY
      DO 126 I=1,ILY
      READ(10,*)Y(I)
 126  CONTINUE
 129  READ(10,*)IEDGEZ
      GOTO (130,135) IEDGEZ
 130  READ (10,*)ILZ,Z1,STEPZ
      Z1=-1.0*Z1
      STEPZ=-1.0*STEPZ
      DO 131 I=1,ILZ
      Z(I)=Z1+(I-1)*STEPZ
 131  CONTINUE
      GOTO 139
 135  READ (10,*)ILZ
      DO 136 I=1,ILZ
      READ(10,*)Z(I)
      Z(I) = -1.0*Z(I)
 136  CONTINUE
```

```
C     READ THE NUMBER OF HEAT SOURCES AND THE POWER DENSITY
C     NOTE-POWER DENSITY IS MULTIPLICATIVE FACTOR USUALLY SET EQUAL TO UNITY
 139  READ(10,*)NSOUR,P0
C     P0 IS THE POWER DENSITY, ASSUMED UNIFORM FOR ALL HEATERS
C     NSOUR IS THE TOTAL NUMBER OF HEATING ELEMENTS ON THE SURFACE OF THE
C     THE TOP LAYER (UP TO 50)
C     THE NEXT LOOP READS IN THE COORDINATES OF THE ORIGIN OF THE
C     HEATING ELEMENTS ALONG WITH THEIR LENGTHS AND WIDTHS
C     THE WEIGHTING FACTOR IS ALSO ENTERED (THIS IS REAL, NONINTEGER)
      DO 140 I=1,NSOUR
      READ(10,*)WTSOUR(I),XSOUR(I), LXSOUR(I),YSOUR(I), LYSOUR(I)
C     WTSOUR(I) IS THE WEIGHTING FACTOR FOR THE I-TH HEATER ELEMENT
C     XSOUR(I) IS THE X COORDINATE OF THE ORIGIN OF I-TH HEATER ELEMENT
C     LXSOUR(I) IS THE LENGTH OF THE I-TH HEATER ALONG THE X DIRECTION
C     YSOUR(I) IS THE Y COORDINATE OF THE ORIGIN OF I-TH HEATER ELEMENT
C     LYSOUR(I) IS THE LENGTH OF THE I-TH HEATER ALONG THE Y DIRECTION
 140  CONTINUE
C     THIS IF...THEN...ELSE IF CONSTRUCTION IS USED TO READ IN THE DATA FOR
C     THE LINES OR AREAS TO BE CONSIDERED IN THE CALCULATION
C     IF THE ANALYSIS IS FOR A POINT FUNCTION, THEN GO OUT OF THE IF.THEN.ELSE
      IF (ITYPE.EQ.1) THEN
           GOTO 170
C     IF THE ANALYSIS IS FOR A LINE AVERAGE, THEN READ THE NUMBER OF LINE
C     SEGMENTS AND THEIR LOCATION
      ELSE IF (ITYPE.EQ.2) THEN
C          READ THE NUMBER OF LINE SEGMENTS
           READ(10,*)NLINE
           DO 150  J=1,NLINE
C          READ THE ORIGIN AND LENGTH OF EACH LINE SEGMENT
C          NOTE THAT THE AVERAGE IS ALONG THE X-DIRECTION FOR GIVEN Y VALUES
C          TO DO THE AVERAGE ALONG THE Y-DIRECTION FOR GIVEN X, SIMPLY ROTATE
C          THE STRUCTURE BY 90 DEGREES AND USE THE CORRESPONDING NEW X'S AND Y'S
C     XLINE(J) IS THE X COORDINATE OF THE ORIGIN OF J-TH LINE ELEMENT
C     LXLINE(J) IS THE LENGTH OF THE J-TH LINE ALONG THE X DIRECTION
           READ(10,*)XLINE(J),LXLINE(J)
 150       CONTINUE
C     IF THE ANALYSIS IS FOR A AREA AVERAGE, THEN READ THE NUMBER OF AREAS
C     AND THEIR LOCATIONS
      ELSE IF (ITYPE.EQ.3) THEN
C          READ THE NUMBER OF AREAS
           READ(10,*)NAREA
           DO 160 J=1,NAREA
C     READ THE FOLLOWING
C     XAREA(J) IS THE X COORDINATE OF THE ORIGIN OF J-TH AREA ELEMENT
C     LXAREA(J) IS THE LENGTH OF THE J-TH AREA ALONG THE X DIRECTION
C     YAREA(J) IS THE Y COORDINATE OF THE ORIGIN OF J-TH AREA ELEMENT
C     LYAREA(J) IS THE LENGTH OF THE J-TH AREA ALONG THE Y DIRECTION
           READ(10,*)XAREA(J),LXAREA(J),YAREA(J),LYAREA(J)
 160       CONTINUE
      ENDIF
 170  CONTINUE
      P04LK = 4.0 * P0 / ( LX* LY* K3)
      PILX = PI /  LX
      PILY = PI /  LY
```

```
C*******************************************************************************
C
C     END OF DATA INPUT SECTION
C     BEGIN CALCULATION OF TEMPERATURE
C     THE SUBROUTINES USED IN THE CALCULATION ARE:
C     1) UZERO(N,M) - CALCULATES THE FOURIER COSINE TRANSFORM OF THE
C        FUNCTION, U(X,Y), THE POWER DENSITY FUNCTION FOR ALL OF THE
C        HEAT SOURCES.
C     2) FUNZ(N,M,Z) - CALCULATES THE Z-DEPENDENT PORTION OF THE SUM
C        REMEMBERING THAT THIS IS A FUNCTION OF THE SUMMATION
C        INDICES (N,M).
C*******************************************************************************
C     CALCULATE THE FOURIER COMPONENTS OF THE HEAT SOURCES, U(N,M)
C*******************************************************************************
      DO 300 MM=1,MUP
      M = MM - 1
      DO 250 NN=1,NUP
         N = NN - 1
         ARUZER(NN,MM)=UZERO(N,M)
 250  CONTINUE
 300  CONTINUE
C*******************************************************************************
C     END OF U(N,M) CALCULATION AND BEGINNING OF MAJOR LOOP FOR Z
C*******************************************************************************
      DO 6000 IZ=1,ILZ
C*******************************************************************************
C     CALCULATE THE Z DEPENDENT PORTION, I.E., FUNZ(N,M,Z)
C*******************************************************************************
      DO 400 MM=1,MUP
      M = MM - 1
      DO 350 NN=1,NUP
         N = NN - 1
         ARFUNZ(NN,MM)=FUNZ(N,M,Z(IZ))*ARUZER(NN,MM)
 350  CONTINUE
 400  CONTINUE
C     THE FOLLOWING IF.THEN.ELSE CONSTRUCTION OPERATES ACCORDING TO THE
C     TYPE OF ANALYSIS TO BE USED.
C     FOR THE POINT FUNCTION ANALYSIS, THE 3000 LOOP IS USED
C     FOR THE LINE AVERAGE ANALYSIS, THE 4000 LOOP IS USED
C     FOR THE AREA AVERAGE ANALYSIS, THE 5000 LOOP IS USED
C     THIS PORTION DOES THE POINT FUNCTION CALCULATION
      IF (ITYPE.EQ.1) THEN
C     BEGINNING OF POINT FUNCTION ANALYSIS
      WRITE(12,2)
      WRITE(12,40)
      WRITE(12,43)
      DO 3000  IY=1,ILY
      DO 3100 MM=1,MUP
         M = MM - 1
         COSYT(MM)=COS(FLOAT(M)*Y(IY)*PILY)
 3100 CONTINUE
```

```
      DO 3000 IX=1,ILX
         SUM=0.0
         DO 3300 MM=1,MUP
         M = MM - 1
         DO 3200 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
            TOP = ARFUNZ(NN,MM) * COS(FLOAT(N)*X(IX)*PILX) * COSYT(MM)
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
 3200    CONTINUE
 3300    CONTINUE
         TEMP = P04LK * SUM
         WRITE (12,44)X(IX),Y(IY),Z(IZ),TEMP
 3000 CONTINUE
C     END OF POINT FUNCTION CALCULATION
      ELSE IF (ITYPE.EQ.2) THEN
C     THIS PORTION DOES THE LINE AVERAGE CALCULATION
      WRITE(12,2)
      WRITE(12,41)
      WRITE(12,45)
      DO 4000  IY=1,ILY
      DO 4010 MM=1,MUP
         M = MM - 1
         COSYT(MM)=COS(FLOAT(M)*Y(IY)*PILY)
 4010 CONTINUE
      DO 4000 J=1,NLINE
         SUM=0.0
         DO 4200 MM=1,MUP
         M = MM - 1
         DO 4100 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
           IF(N.EQ.0) GO TO 4160
           TERMX=SIN(FLOAT(N)*PI*(XLINE(J)+LXLINE(J))/LX)
     1           -SIN(FLOAT(N)*PI*XLINE(J)/LX)
           TERMX=TERMX*LX/(FLOAT(N)*PI)
           GO TO 4165
 4160   TERMX=LXLINE(J)
 4165   CONTINUE
            TOP = ARFUNZ(NN,MM) * COSYT(MM) * TERMX
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
 4100    CONTINUE
         TEMP = P04LK * SUM / LXLINE(J)
 4200    CONTINUE
      WRITE(12,46)J,XLINE(J),LXLINE(J),Y(IY),Z(IZ),TEMP
 4000 CONTINUE
```

```
C     END OF LINE AVERAGE PORTION OF THE CODE
      ELSE IF (ITYPE.EQ.3) THEN
C     THIS PART PERFORMS THE AREA AVERAGE CALCULATION
      WRITE(12,2)
      WRITE(12,42)
      WRITE(12,47)
      DO 5000 J=1,NAREA
         SUM=0.0
         DO 5100 MM=1,MUP
         M = MM - 1
         DO 5200 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
      AREA=0.0
         IF(N.EQ.0) GO TO 5160
           TERMX = SIN(FLOAT(N)*PI*(XAREA(J) +  LXAREA(J))/ LX)
     1             - SIN(FLOAT(N)*PI*XAREA(J)/ LX)
           TERMX=TERMX* LX/(FLOAT(N)*PI)
           GO TO 5165
 5160      TERMX= LXAREA(J)
 5165      IF(M.EQ.0) GO TO 5164
           TERMY = SIN(FLOAT(M)*PI*(YAREA(J) +  LYAREA(J))/ LY)
     1             - SIN(FLOAT(M)*PI*YAREA(J)/ LY)
           TERMY=TERMY* LY/(FLOAT(M)*PI)
           GO TO 5166
 5164      TERMY= LYAREA(J)
 5166      TERMI=TERMX*TERMY
         AREA=TERMI
            TOP = ARFUNZ(NN,MM) * AREA
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
 5200    CONTINUE
         TEMP = P04LK * SUM / (LXAREA(J)*LYAREA(J))
 5100    CONTINUE
      WRITE(12,48)J,XAREA(J),LXAREA(J),YAREA(J),LYAREA(J),Z(IZ),TEMP
 5000 CONTINUE
      ENDIF
 6000 CONTINUE
      WRITE(12,27) LX, LY
      WRITE(12,3)
      WRITE(12,5) L3, L2, L1
      WRITE(12,4) K3, K2, K1
      WRITE(12,6)NUP,MUP
      WRITE(12,7)NSOUR
      WRITE(12,11) P0
      WRITE(12,8)
      WRITE(12,9)
      DO 3888 I=1,NSOUR
      WRITE(12,10)I,WTSOUR(I),XSOUR(I),YSOUR(I), LXSOUR(I), LYSOUR(I)
 3888 CONTINUE
```

```
      IF (ITYPE.EQ.1) THEN
         GOTO 7000
      ELSE IF (ITYPE.EQ.2) THEN
       WRITE(12,32) NLINE
       DO 6100 J=1,NLINE
       WRITE(12,33)J,XLINE(J),LXLINE(J)
 6100  CONTINUE
      ELSE IF (ITYPE.EQ.3) THEN
       WRITE(12,34)NAREA
       DO 6200 J=1,NAREA
       WRITE(12,35)J,XAREA(J),LXAREA(J),YAREA(J),LYAREA(J)
 6200  CONTINUE
      ENDIF
      GO TO 7000
 3999 WRITE(12,88)
 7000 STOP
      END
C*******************************************************************************
C     END OF THE MAIN PROGRAM
```

```
C     FUNCTION UZERO LISTING
C*************************************************************************
C     DESCRIPTION OF THE FUNCTION UZERO(N,M)
C     THIS FUNCTION CALCULATES THE DOUBLE FOURIER COSINE TRANSFORM
C     OF THE POWER DENSITY FUNCTION, U(X,Y).  THIS IS THE TRANSFORM
C     FOR ALL OF THE HEAT SOURCES.  THE ASSUMPTION IS MADE THAT THE
C     POWER DENSITY IS UNIFORM AND EQUAL TO UNITY OVER THE SURFACE
C     OF THE HEATING ELEMENTS.  THAT IS,
C       U(X,Y)=1  (XSOUR(I)<=X<=XSOUR(I)+LXSOUR(I)  AND
C                  YSOUR(I)<=Y<=YXOUR(I)+LYSOUR(I) ).
C       U(X,Y)=0  OTHERWISE.
C     UNDER THESE CONDITIONS, IT IS POSSIBLE TO ANALYTICALLY EVALUATE
C     THE DOUBLE INTEGRAL FOR EACH HEATING ELEMENT.  AS THE HEATING
C     ELEMENTS ARE ASSUMED TO BE INDEPENDENT, THE CONTRIBUTION FROM
C     EACH ELEMENT MAY BE ADDED TO OBTAIN THE U(N,M) FOR ALL.
C
C     NONUNIFORM POWER DENSITIES MAY BE TAKEN CARE OF BY USING THE
C     WEIGHTING FACTOR FOR EACH ELEMENT (READ IN THE MAIN PROGRAM)
C*************************************************************************
      FUNCTION UZERO(N,M)
      DIMENSION WTSOUR(50), XSOUR(50), YSOUR(50)
      REAL LXSOUR(50), LYSOUR(50), K3, K2, K1, LX, LY, L3, L2, L1
      COMMON  K3, K2, K1, LX, LY, L3, L2, L1
      COMMON NSOUR,WTSOUR,XSOUR,YSOUR, LXSOUR, LYSOUR
      PI=3.14159265
      UZERO=0.0
      DO 500 I=1,NSOUR
         IF(N.EQ.0) GO TO 100
           TERMX = SIN(FLOAT(N)*PI*(XSOUR(I) +  LXSOUR(I))/ LX)
     1             - SIN(FLOAT(N)*PI*XSOUR(I)/ LX)
           TERMX=TERMX* LX/(FLOAT(N)*PI)
           GO TO 150
  100    TERMX= LXSOUR(I)
  150    IF(M.EQ.0) GO TO 200
           TERMY = SIN(FLOAT(M)*PI*(YSOUR(I) +  LYSOUR(I))/ LY)
     1             - SIN(FLOAT(M)*PI*YSOUR(I)/ LY)
           TERMY=TERMY* LY/(FLOAT(M)*PI)
           GO TO 250
  200    TERMY= LYSOUR(I)
  250    TERMI=TERMX*TERMY
         UZERO=UZERO+TERMI*WTSOUR(I)
  500  CONTINUE
       RETURN
       END
```

APPENDIX A - TXYZ30 LISTING

```
C*******************************************************************************
C     FUNCTION FUNZ LISTING
C
C     DESCRIPTION OF THE FUNCTION FUNZ(N,M,Z)
C     THIS FUNCTION IS USED TO CALCULATE THE Z DEPENDENT PART OF THE
C     FUNCTION USING THE S=0 VERSIONS OF EQUATIONS (15)-(17) OF KOKKAS.
C     THESE ARE USED IN CONJUNCTION WITH EQUATIONS (18)-(22) ALSO FOR
C     S=0 (STEADY-STATE CONDITION).  THE SPECIFIC FORM OF FUNZ IS
C     DETERMINED BY THE VALUE OF Z, I.E., IF Z FALLS IN THE TOP, MIDDLE,
C     OR BOTTOM LAYER OF THE STRUCTURE.  IN ADDITION, SPECIAL CARE IS
C     TAKEN TO EVALUATE FUNZ FOR THE CASE WHERE GAMMA=0 AS THESE ARE
C     SIMPLE, BUT THE COMPUTER DOES NOT KNOW HOW TO EVALUATE LIMITS.
C     IN ADDITION, THE FORM OF THE THREE SOLUTIONS HAS BEEN CHANGED TO
C     GET RID OF THE ARTIFICIAL OVERFLOW PROBLEMS COMING FROM THE
C     HYPERBOLIC FUNCTIONS, COSH AND SINH, FOR THE CASES WHERE THE
C     ARGUMENTS BECOME LARGE.
C
C*******************************************************************************
      FUNCTION FUNZ(N,M,Z)
      DIMENSION WTSOUR(50), XSOUR(50), YSOUR(50)
      REAL LXSOUR(50), LYSOUR(50), K3, K2, K1, LX, LY, L3, L2, L1
      COMMON  K3, K2, K1, LX, LY, L3, L2, L1
      COMMON NSOUR,WTSOUR,XSOUR,YSOUR, LXSOUR, LYSOUR
      PI=3.14159265
      GAMMA=SQRT((FLOAT(N)*PI/ LX)**2 + (FLOAT(M)*PI/ LY)**2)
      VS=GAMMA* L3
      VC=GAMMA* L2
      VI=GAMMA* L1
      VT=GAMMA*( L3+Z)
      VM=GAMMA*( L3+ L2+Z)
      VB=GAMMA*( L3+ L2+ L1+Z)
      BOT1=TANH(VS)*TANH(VI)
      BOT1=BOT1+( K1/ K2)*TANH(VS)*TANH(VC)
      BOT2=( K2/ K3)*TANH(VI)*TANH(VC)+( K1/ K3)
      GFUNC=1.0/(BOT1+BOT2)
      AZ=ABS(Z)
      IF (AZ.GT. L3) GO TO 500
```

```
C*************************************************************************
C            TOP LAYER CALCULATION
C     THIS PORTION IS THE TOP LAYER CALCULATION WHICH IS DEFAULTED
C     TO IF Z FALLS INTO THE TOP LAYER
C*************************************************************************
      IF (GAMMA.EQ.0.0) GO TO 100
      TERMS1=TANH(VI)+( K1/ K2)*TANH(VC)
      TERMS2=( K2/ K3)*TANH(VT)*(TANH(VI)*TANH(VC)+( K1/ K2))
      TERMS=TERMS1+TERMS2
      IF (Z.EQ.0.0) GO TO 90
      IF (VS.GT.5.0.AND.VT.GT.5.0) GO TO 80
      IF (VS.LT.5.0) GO TO 10
      C1=2.0*EXP(-VS)
      GO TO 20
 10   C1=1.0/COSH(VS)
 20   CONTINUE
      IF (VT.LT.5.0) GO TO 30
      C2=0.5*EXP(VT)
      GO TO 40
 30   C2=COSH(VT)
 40   CONTINUE
      FUNZ=GFUNC*TERMS*C1*C2/GAMMA
      RETURN
 80   FUNZ=GFUNC*TERMS*EXP(GAMMA*Z)/GAMMA
      RETURN
 90   FUNZ=GFUNC*TERMS/GAMMA
      RETURN
 100  FUNZ=( L3+Z)+( K3/ K2)* L2+( K3/ K1)* L1
      RETURN
 500  TOTAL= L3+ L2
      IF (AZ.GT.TOTAL) GO TO 1500
```

```
C*************************************************************************
C            MIDDLE LAYER CALCULATION
C     THIS IS THE MIDDLE LAYER CALCULATION WHICH IS DEFAULTED TO IF
C     Z FALLS INTO THIS DOMAIN OF DEPTHS
C*************************************************************************
      IF (GAMMA.EQ.0.0) GO TO 1000
      TERMC=TANH(VI)+( K1/ K2)*TANH(VM)
      IF (VS.GT.5.0.AND.VC.GT.5.0.AND.VM.GT.5.0) GO TO 800
      IF (VS.LT.5.0.AND.VC.GT.5.0.AND.VM.GT.5.0) GO TO 900
      IF (VS.LT.5.0) GO TO 250
      C1=2.0*EXP(-VS)
      GO TO 260
 250  C1=1.0/COSH(VS)
 260  CONTINUE
      IF (VC.LT.5.0) GO TO 270
      C2=2.0*EXP(-VC)
      GO TO 280
 270  C2=1.0/COSH(VC)
 280  CONTINUE
      IF (VM.LT.5.0) GO TO 320
      C3=0.5*EXP(VM)
      GO TO 330
 320  C3=COSH(VM)
 330  CONTINUE
      FUNZ=GFUNC*TERMC*C1*C2*C3/GAMMA
      RETURN
 800  FUNZ=GFUNC*TERMC*2.0*EXP(GAMMA*Z)/GAMMA
      RETURN
 900  FUNZ=GFUNC*TERMC*EXP(VS+GAMMA*Z)/(COSH(VS)*GAMMA)
      RETURN
 1000 FUNZ=( K3/ K1)* L1+( K3/ K2)*( L3+ L2+Z)
      RETURN
```

```
C*****************************************************************************
C           BOTTOM LAYER CALCULATION
C     THIS IS THE BOTTOM LAYER CALCULATION WHICH IS USED IF Z FALLS
C     INTO THE BOTTOM LAYER
C*****************************************************************************
 1500 IF (GAMMA.EQ.0.0) GO TO 2000
      IF (VS.GT.5.0.AND.VC.GT.5.0.AND.VI.GT.5.0.AND.
     1 VB.GT.5.0) GO TO 1900
      IF(VB.GT.5.0.AND.VS.GT.5.0.AND.VC.LT.5.0.AND.VI.LT.5.0)GO TO 2100
      IF(VB.GT.5.0.AND.VS.LT.5.0.AND.VC.GT.5.0.AND.VI.LT.5.0)GO TO 2200
      IF(VB.GT.5.0.AND.VS.LT.5.0.AND.VC.LT.5.0.AND.VI.GT.5.0)GO TO 2300
      IF(VB.GT.5.0.AND.VS.GT.5.0.AND.VC.GT.5.0.AND.VI.LT.5.0)GO TO 2400
      IF(VB.GT.5.0.AND.VS.GT.5.0.AND.VC.LT.5.0.AND.VI.GT.5.0)GO TO 2500
      IF(VB.GT.5.0.AND.VS.LT.5.0.AND.VC.GT.5.0.AND.VI.GT.5.0)GO TO 2600
      IF (VS.LT.5.0) GO TO 1550
      C1=2.0*EXP(-VS)
      GO TO 1560
 1550 C1=1.0/COSH(VS)
 1560 CONTINUE
      IF (VC.LT.5.0) GO TO 1570
      C2=2.0*EXP(-VC)
      GO TO 1580
 1570 C2=1.0/COSH(VC)
 1580 CONTINUE
      IF (VI.LT.5.0) GO TO 1590
      C3=2.0*EXP(-VI)
      GO TO 1600
 1590 C3=1.0/COSH(VI)
 1600 CONTINUE
      C4=SINH(VB)
      FUNZ=GFUNC*C1*C2*C3*C4/GAMMA
      RETURN
 1900 FUNZ=GFUNC*4.0*EXP(GAMMA*Z)/GAMMA
      RETURN
 2000 FUNZ=( K3/ K1)*( L3+ L2+ L1+Z)
      RETURN
 2100 FUNZ=GFUNC*EXP(VB-VS)/(GAMMA*COSH(VC)*COSH(VI))
      RETURN
 2200 FUNZ=GFUNC*EXP(VB-VC)/(GAMMA*COSH(VS)*COSH(VI))
      RETURN
 2300 FUNZ=GFUNC*EXP(VB-VI)/(GAMMA*COSH(VS)*COSH(VC))
      RETURN
 2400 FUNZ=GFUNC*2.0*EXP(VB-VS-VC)/(GAMMA*COSH(VI))
      RETURN
 2500 FUNZ=GFUNC*2.0*EXP(VB-VS-VI)/(GAMMA*COSH(VC))
      RETURN
 2600 FUNZ=GFUNC*2.0*EXP(VB-VC-VI)/(GAMMA*COSH(VS))
      RETURN
      END
```

APPENDIX B - TML LISTING

```
C*************************************************************************
C     TML VERSION 1.0 - VERSION DATE 06/27/95
C
C     THIS IS THE THERMAL MULTILAYER PROGRAM WHICH IS BASED UPON A THERMAL
C     RECURSION RELATION WHICH IS SIMILAR TO THAT USED IN THE SOLUTION OF
C     THE LAPLACE EQUATION FOR TWO-PROBE AND FOUR-PROBE RESISTANCE ANALYSIS.
C     THIS PROGRAM CALCULATES THE STEADY-STATE SURFACE TEMPERATURE FOR A
C     MULTILAYER RECTANGULAR STRUCTURE WITH AN ARBITRARY NUMBER OF LAYERS.
C     THE CALCULATION IS PERFORMED FOR AN ARBITRARY NUMBER OF RECTANGULAR
C     HEAT SOURCES/SINKS ON THE TOP SURFACE.
C     THE TEMPERATURE MAY BE CALCULATED AS A POINT FUNCTION (X,Y), A LINE
C     AVERAGE, OR AN AREA AVERAGE ON THE TOP SURFACE.
C     THE CALCULATION FOLLOWS FROM THE INPUT OF THE THICKNESSES AND THERMAL
C     CONDUCTIVITIES OF ALL OF THE LAYERS IN THE STRUCTURE.
C     IT IS IMPORTANT TO EMPHASIZE THAT THE CALCULATION IS GENERAL FOR THE
C     MULTILAYER STRUCTURE AND THE APPLICATION TO SEMICONDUCTOR STRUCTURES
C     IS A SPECIAL CASE.
C
C     THE STARTING POINT IS GIVEN IN EQUATIONS (13)-(23), WITH S=0
C     (ZERO FREQUENCY, STEADY-STATE CONDITION), IN THE PAPER BY KOKKAS.
C
C     REFERENCES: THE ORIGINAL MATHEMATICAL ANALYSIS OF THE THREE-LAYER
C                 STRUCTURE WAS PERFORMED IN THE PAPER "THERMAL ANALYSIS
C                 OF MULTIPLE-LAYERED STRUCTURES" BY ACHILLES G. KOKKAS,
C                 IEEE TRANS. ELEC. DEV. VOL. ED-21, NO. 11, 674-681 (1974).
C                 THIS PAPER WAS DRAWN FROM HIS PHD THESIS: A. G. KOKKAS,
C                 "ANALYSIS AND DESIGN OF ELECTROTHERMAL INTEGRATED CIRCUITS,"
C                 PH.D. THESIS, MIT, 1972.
C
C                 THE ORIGINAL FORTRAN IMPLEMENTATION OF THE STEADY STATE
C                 KOKKAS EQUATIONS IS CONTAINED IN THE TXYZ CODE AND
C                 WAS PRESENTED IN THE REPORT "SEMICONDUCTOR MEASUREMENT
C                 TECHNOLOGY: TXYZ: A PROGRAM FOR SEMICONDUCTOR IC THERMAL
C                 ANALYSIS"  BY JOHN ALBERS, NBS SPECIAL PUBLICATION 400-76
C                 (APRIL 1984).
C
C                 VERSION 2.0 OF THE TXYZ CODE WAS PRESENTED IN THE REPORT
C                 "SEMICONDUCTOR MEASUREMENT TECHNOLOGY: VERSION 2.0 OF THE
C                 TXYZ THERMAL ANALYSIS PROGRAM: TXYZ20" BY JOHN ALBERS,
C                 NIST SPECIAL PUBLICATION 400-89 (JUNE 1992).
C
C                 VERSION 3.0 OF THE TXYZ CODE AND THE ACCOMPANYING THERMAL
C                 MULTILAYER CODES ARE DISCUSSED IN THIS REPORT
C                 "SEMICONDUCTOR MEASUREMENT TECHNOLOGY: HOTPAC: PROGRAMS
C                 FOR THERMAL ANALYSIS INCLUDING VERSION 3.0 OF THE TXYZ
C                 PROGRAM, TXYZ30, AND THE THERMAL MULTILAYER PROGRAM, TML"
C                 BY JOHN ALBERS, NIST SPECIAL PUBLICATION 400-96.
C
C                 THE REVIEW AND APPLICATION OF THE RECURSION RELATION
C                 TECHNIQUE FOR THE ANALYSIS OF THE LAPLACE EQUATION USED IN
C                 TWO-PROBE AND FOUR-PROBE RESISTANCE ARE CONTAINED IN THE
C                 REPORT "SEMICONDUCTOR MEASUREMENT TECHNOLOGY: A COLLECTION
C                 OF COMPUTER PROGRAMS FOR TWO-PROBE RESISTANCE (SPREADING
C                 RESISTANCE) AND FOUR-PROBE RESISTANCE CALCULATIONS, RESPAC
C                 BY JOHN ALBERS AND HARRY L. BERKOWITZ, NIST SPECIAL
C                 PUBLICATION 400-91, 1993.
C
```

APPENDIX B - TML LISTING

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C                THE THERMAL RECURSION RELATION TECHNIQUE HAS BEEN DISCUSSED
C                IN THE PAPERS: "AN EXACT SOLUTION OF THE STEADY-STATE
C                SURFACE TEMPERATURE FOR A GENERAL MULTILAYER STRUCTURE"
C                BY JOHN ALBERS, PROCEEDINGS TENTH IEEE SEMI-THERM SYMPOSIUM,
C                PP. 129-137, 1994 AND
C                "AN EXACT RECURSION RELATION SOLUTION FOR THE STEADY-STATE
C                SURFACE TEMPERATURE OF A GENERAL MULTILAYER STRUCTURE"
C                BY JOHN ALBERS, IEEE TRANSACTIONS ON COMPONENTS, PACKAGING
C                AND MANUFACTURING TECHNOLOGY - PART A, VOL. 18, NO. 1,
C                PP. 31-38, 1995.
C
C*************************************************************************
C     IN THE PRESENT FORM, THE PROGRAM ALLOWS UP TO 500 TERMS TO BE INCLUDED
C     IN BOTH THE N SUM (ALONG X) AND THE M SUM (ALONG Y).  TO GO BEYOND
C     THIS NUMBER, SUBSTITUTE THE FOLLOWING TWO LINES WITH THE APPROPRIATE
C     VALUES OF NX AND MY FOR THE FIRST TWO DIMENSION STATEMENTS - MAKE SURE TO
C     REMOVE THE COMMENTS FROM THE NEW LINES AND COMMENT OUT THE REPLACED LINES.
C      DIMENSION X(100), Y(100), Z(100), COSYT(MY)
C      DIMENSION ARUZER(NX,MY), ARFUNZ(NX,MY)
C     ALSO REPLACE THE TESTING LINE ABOVE THE LINE LABELLED 110 WITH
C      IF (NUP.GT.NX.OR.MUP.GT.MY) GO TO 3999
      DIMENSION X(100), Y(100), COSYT(500)
      DIMENSION ARUZER(500,500), ARFUN0(500,500)
      DIMENSION WTSOUR(50), XSOUR(50), YSOUR(50)
      REAL LXSOUR(50), LYSOUR(50), K(20), LX, LY, L(20)
      DIMENSION XLINE(30),XAREA(30),YAREA(30)
      REAL LXLINE(30),LXAREA(30),LYAREA(30)
      COMMON  LX, LY, NLAY, K, L
      COMMON NSOUR,WTSOUR,XSOUR,YSOUR, LXSOUR, LYSOUR
      PI=3.14159265
C     INPUT DATA IS READ FROM 10 AND OUTPUT GOES TO 12. THESE ARE WRITTEN IN
C     LOWER CASE.  MANY OPERATING SYSTEMS TAKE UPPER AND LOWER CASE AS
C     EQUIVALENT.  THE UNIX OPERATING SYSTEM VIEWS THE UPPER AND LOWER
C     CASE-NAMED FILES AS DIFFERENT FILES.
C     UNIX DEFAULTS TO THE LOWER CASE WHICH IS USED HERE.
      open( unit=10, file='input.dat', status = 'unknown')
      open( unit=12, file='output.dat', status = 'unknown' )
 2    FORMAT(1X,'STEADY-STATE THERMAL MULTILAYER CALCULATION'/
     1' USING THE THERMAL RECURSION RELATION IN KOKKAS EQUATIONS')
 3    FORMAT(1X,'THICKNESSES AND THERMAL CONDUCTIVITIES OF LAYERS')
 4    FORMAT(1X,'NUMBER OF LAYERS IN STRUCTURE=',I2)
 5    FORMAT(1X,'L',I2,'= ',F10.5,'  K',I2,'= ',F10.5)
 6    FORMAT(1X,'UPPER SUMMATION LIMITS ',2X,' NUP=',I5,
     1 '  MUP=',I5)
 7    FORMAT(1X,'NUMBER OF HEAT SOURCES=',I5)
 8    FORMAT(1X,'WEIGHTS, COORDINATES, LENGTHS, WIDTHS OF SOURCES')
 9    FORMAT(1X,'HEAT SOURCE',4X,'WTSOUR',8X,'XSOUR',8X,'YSOUR',7X,
     1 'LXSOUR',7X,'LYSOUR')
 10   FORMAT(4X,I3,5X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
 11   FORMAT(1X,'POWER DENSITY=',F11.6)
 27   FORMAT(1X,'LX= ',F7.2,3X,' LY= ',F7.2)
 22   FORMAT(1X,F12.4,2X,F12.4,2X,F12.4,2X,F12.4)
 31   FORMAT(1X,6I7)
 32   FORMAT(1X,'NUMBER OF LINES=',I3)
 33   FORMAT(1X,I3,3X,F10.5,3X,F10.5)
 34   FORMAT(1X,'NUMBER OF AREAS=',I3)
```

```
 35   FORMAT(1X,I3,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
 40   FORMAT(1X,'POINT FUNCTION EVALUATION OF SURFACE T(X,Y)')
 41   FORMAT(1X,'LINE AVERAGE EVALUATION OF SURFACE TEMPERATURE')
 42   FORMAT(1X,'AREA AVERAGE EVALUATION OF SURFACE TEMPERATURE')
 43   FORMAT(6X,'X',12X,'Y',11X,'T(X,Y)')
 44   FORMAT(1X,F10.5,3X,F10.5,3X,F10.5)
 45   FORMAT(1X,'LINE #',5X,'XLINE',7X,'LXLINE',8X,'Y',9X,'AVE TEMP')
 46   FORMAT(1X,I3,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
 47   FORMAT(1X,'AREA #',5X,'XAREA',7X,'LXAREA',8X,'YAREA',7X,
     1 'LYAREA',3X,'AVE TEMP')
 48   FORMAT(1X,I3,3X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5,1X,F10.5)
 51   FORMAT(1X,I4,3X,I4)
 52   FORMAT(1X,F10.5,3X,F10.5)
 53   FORMAT(1X,I1)
 54   FORMAT(1X,I4,3X,F10.5,3X,F10.5)
 55   FORMAT(1X,F10.5)
 56   FORMAT(1X,I2,3X,F10.5)
 57   FORMAT(1X,F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
 88   FORMAT(1X,'YOUR UPPER LIMIT OF SUMMATION IS TOO LARGE. TRY AGAIN')
C*******************************************************************************
C
C                    -----INPUT SECTION-----
C     THE FOLLOWING VARIABLES ARE READ BY THE PROGRAM FROM FOR010
C
C         ITYPE-----TYPE OF ANALYSIS
C                   =1 FOR POINT FUNCTION, T(XY,0), EVALUATION
C                   =2 FOR LINE AVERAGE, <T(Y,0)>, EVALUATION
C                   =3 FOR AREA AVERAGE, <T(0)>, EVALUATION
C
C         LX--------X DIMENSION OF THE MULTILAYER STRUCTURE
C         LY--------Y DIMENSION OF THE MULTILAYER STRUCTURE
C         NLAY------NUMBER OF LAYERS IN THE STRUCTURE (UP TO 20 LAYERS
C                   BUT THE CODE CAN GO BEYOND THIS BY EDITING THE
C                   APPROPRIATE DIMENSION STATEMENTS IN THE MAIN PROGRAM
C                   AND SUBROUTINES/FUNCTIONS)
C
C            THE NEXT NLAY LINES CONTAIN THE THICKNESSES AND THERMAL
C            CONDUCTIVITIES OF THE N-LAYERS BEGINNING AT THE SURFACE AND GOING
C            TO THE BOTTOM LAYER (CODE TAKES CARE OF NUMBERING FROM NLAY,...,1)
C
C         L(I), K(I)-THICKNESS AND THERMAL CONDUCTIVITY OF I-TH LAYER
C         NUP-------UPPER LIMIT OF N SUM, X DIRECTION
C         MUP-------UPPER LIMIT OF M SUM, Y DIRECTION
C
C         IEDGEX----INDEX FOR GENERATING THE VALUES OF X TO BE USED
C                    =1 TO READ DATA FOR FIXED INCREMENT X VALUES
C                    =2 TO READ IN ARRAY OF X VALUES OF NONFIXED INCREMENT)
C                   IF IEDGEX=1 THEN READ THE THREE VARIABLES (ON SAME LINE)
C                       ILX  THE NUMBER OF X VALUES TO BE USED
C                       X1   THE VALUE OF THE FIRST POINT IN X
C                       STEPX (THE INCREMENT IN X)
C                   IF IEDGEX=2 THEN READ THE VARIABLE AND ARRAY (ONE PER LINE)
C                       ILX  THE NUMBER OF X VALUES TO BE USED
C                       X(I) THE ARRAY OF X VALUES (I=1,ILX)
C
```

```
C          IEDGEY----INDEX FOR GENERATING THE VALUES OF Y TO BE USED
C                     =1 TO READ DATA FOR FIXED INCREMENT Y VALUES
C                     =2 TO READ IN ARRAY OF X VALUES OF NONFIXED INCREMENT)
C                    IF IEDGEY=1 THEN READ THE THREE VARIABLES (ON SAME LINE)
C                        ILY  THE NUMBER OF Y VALUES TO BE USED
C                        Y1   THE VALUE OF THE FIRST POINT IN Y
C                        STEPY (THE INCREMENT IN Y)
C                    IF IEDGEY=2 THEN READ THE VARIABLE AND ARRAY (ONE PER LINE)
C                        ILY  THE NUMBER OF Y VALUES TO BE USED
C                        Y(I) THE ARRAY OF Y VALUES (I=1,ILY)
C
C         NOTE: THERE IS NO INPUT FOR THE Z VALUES AS Z=0 THROUGHOUT THE
C               CALCULATION
C
C         IMPORTANT: THE POINT FUNCTION EVALUATION OF THE TEMPERATURE IS
C                    THE MOST ELEMENTAL VERSION.  IN ORDER TO SIMPLIFY THE
C                    CONSTRUCTION OF THE DATA FILES FOR LINE AND AREA
C                    AVERAGES, THE PROGRAM EXPECTS TO SEE THE ABOVE IEDGEX,
C                    AND IEDGEY DATA.  THIS IS READ EVEN IF IT IS NOT
C                    USED FOR THE AVERAGE VERSIONS.  HOWEVER, THE LINE AND
C                    AREA INFORMATION MAY THEN BE SIMPLY APPENDED TO THE END
C                    OF THE DATA FILE IN ORDER TO RUN THESE VERSIONS.  SEE THE
C                    SAMPLE I/O FILES FOR AN EXAMPLE OF THIS.
C
C          NSOUR-----NUMBER OF HEAT SOURCES (UP TO 50)
C          P0--------POWER DENSITY
C
C          THE NEXT NSOUR LINES READ THE FOLLOWING INFORMATION FOR THE
C          HEAT SOURCES (WITH ALL THE INFORMATION FOR EACH OF THE ELEMENTS
C                        ON A SINGLE LINE)
C
C          WTSOUR(I)-WEIGHTING FACTOR OF I-TH SOURCE
C                    (POSITIVE FOR SOURCE, NEGATIVE FOR SINK)
C          XSOUR(I)--X COORDINATE OF ORIGIN OF I-TH SOURCE
C          LXSOUR(I)-LENGTH ALONG X DIRECTION OF I-TH SOURCE
C          YSOUR(I)--Y COORDINATE OF ORIGIN OF I-TH SOURCE
C          LYSOUR(I)-LENGTH ALONG Y DIRECTION OF I-TH SOURCE
C
C          IF ITYPE=1, THE POINT FUNCTION CALCULATION CONTINUES WITH THE ABOVE
C                      SET OF X,Y VALUES
C          IF ITYPE=2, THE LINE AVERAGE CALCULATION READS THE FOLLOWING:
C                      NLINE---THE NUMBER OF LINE SEGMENTS TO DO THE AVERAGE
C                      THE NEXT NLINE LINES THEN READ
C                      XLINE(J), LXLINE(J)--THE LOCATION AND LENGTH OF THE
C                                           J-TH LINE ELEMENT
C          IF ITYPE=3, THE AREA AVERAGE CALCULATION READS THE FOLLOWING:
C                      NAREA---THE NUMBER OF AREA SEGMENTS TO DO THE AVERAGE
C                      THE NEXT NAREA LINES THEN READ
C                      XAREA(J),LXAREA(J),YAREA(J),LYAREA(J)-THE LOCATION
C                      AND LENGTHS OF THE J-TH AREA ELMENT
C*************************************************************************
```

```
C     READ ITYPE
      READ(10,*) ITYPE
C     READ LX AND LY (THE X AND Y DIMENSIONS OF THE RECTANGULAR STRUCTURE)
      READ(10,*) LX, LY
C     READ THE NUMBER OF LAYERS IN THE STRUCTURE
      READ(10,*) NLAY
C     READ LAYER THICKNESS AND THERMAL CONDUCTIVITY FROM SURFACE TO BOTTOM
      DO 101 I=NLAY,1,-1
      READ(10,*) L(I),K(I)
 101  CONTINUE
C     READ NUP AND MUP (UPPER LIMIT OF THE SUMMATION OVER THE INDEX N (X-DIR)
C                       UPPER LIMIT OF THE SUMMATION OVER THE INDEX M (Y-DIR))
C     NUP AND MUP MUST BE LESS THAN OR EQUAL TO THE DIMENSIONALITY OF
      READ(10,*)NUP,MUP
      IF (NUP.GT.500.OR.MUP.GT.500) GO TO 3999
      READ(10,*)IEDGEX
      GOTO (110,115)IEDGEX
 110  READ (10,*)ILX,X1,STEPX
      DO 111 I=1,ILX
      X(I)=X1+(I-1)*STEPX
 111  CONTINUE
      GOTO 119
 115  READ (10,*)ILX
      DO 116 I=1,ILX
      READ(10,*)X(I)
 116  CONTINUE
 119  READ(10,*)IEDGEY
      GOTO (120,125) IEDGEY
 120  READ (10,*)ILY,Y1,STEPY
      DO 121 I=1,ILY
      Y(I)=Y1+(I-1)*STEPY
 121  CONTINUE
      GOTO 129
 125  READ (10,*)ILY
      DO 126 I=1,ILY
      READ(10,*)Y(I)
 126  CONTINUE
 129  Z=0.0
C     READ THE NUMBER OF HEAT SOURCES AND THE POWER DENSITY
C     NOTE-POWER DENSITY IS MULTIPLICATIVE FACTOR USUALLY SET EQUAL TO UNITY
C     P0 IS THE POWER DENSITY, ASSUMED UNIFORM FOR ALL HEATERS
C     NSOUR IS THE TOTAL NUMBER OF HEATING ELEMENTS ON THE SURFACE OF THE
C     THE TOP LAYER (UP TO 50)
 139  READ(10,*)NSOUR,P0
C     THE NEXT LOOP READS IN THE COORDINATES OF THE ORIGIN OF THE
C     HEATING ELEMENTS ALONG WITH THEIR LENGTHS AND WIDTHS
C     THE WEIGHTING FACTOR IS ALSO ENTERED (THIS IS REAL, NONINTEGER)
C     WTSOUR(I) IS THE WEIGHTING FACTOR FOR THE I-TH HEATER ELEMENT
C     XSOUR(I) IS THE X COORDINATE OF THE ORIGIN OF I-TH HEATER ELEMENT
C     LXSOUR(I) IS THE LENGTH OF THE I-TH HEATER ALONG THE X DIRECTION
C     YSOUR(I) IS THE Y COORDINATE OF THE ORIGIN OF I-TH HEATER ELEMENT
C     LYSOUR(I) IS THE LENGTH OF THE I-TH HEATER ALONG THE Y DIRECTION
      DO 140 I=1,NSOUR
      READ(10,*)WTSOUR(I),XSOUR(I), LXSOUR(I),YSOUR(I), LYSOUR(I)
 140  CONTINUE
```

```
C     THIS IF...THEN...ELSE IF CONSTRUCTION IS USED TO READ IN THE DATA FOR
C     THE LINES OR AREAS TO BE CONSIDERED IN THE CALCULATION
C     IF THE ANALYSIS IS FOR A POINT FUNCTION, THEN GO OUT OF THE IF.THEN.ELSE
      IF (ITYPE.EQ.1) THEN
           GOTO 170
C     IF THE ANALYSIS IS FOR A LINE AVERAGE, THEN READ THE NUMBER OF LINE
C     SEGMENTS AND THEIR LOCATION
      ELSE IF (ITYPE.EQ.2) THEN
C          READ THE NUMBER OF LINE SEGMENTS
           READ(10,*)NLINE
           DO 150  J=1,NLINE
C          READ THE ORIGIN AND LENGTH OF EACH LINE SEGMENT
C          NOTE THAT THE AVERAGE IS ALONG THE X-DIRECTION FOR GIVEN Y VALUES
C          TO DO THE AVERAGE ALONG THE Y-DIRECTION FOR GIVEN X, SIMPLY ROTATE
C          THE STRUCTURE BY 90 DEGREES AND USE THE CORRESPONDING NEW X'S AND Y'S
C     XLINE(J) IS THE X COORDINATE OF THE ORIGIN OF J-TH LINE ELEMENT
C     LXLINE(J) IS THE LENGTH OF THE J-TH LINE ALONG THE X DIRECTION
           READ(10,*)XLINE(J),LXLINE(J)
 150       CONTINUE
C     IF THE ANALYSIS IS FOR AN AREA AVERAGE, THEN READ THE NUMBER OF AREAS
C     AND THEIR LOCATIONS
      ELSE IF (ITYPE.EQ.3) THEN
C          READ THE NUMBER OF AREAS
           READ(10,*)NAREA
           DO 160 J=1,NAREA
C     READ THE FOLLOWING
C     XAREA(J) IS THE X COORDINATE OF THE ORIGIN OF J-TH AREA ELEMENT
C     LXAREA(J) IS THE LENGTH OF THE J-TH AREA ALONG THE X DIRECTION
C     YAREA(J) IS THE Y COORDINATE OF THE ORIGIN OF J-TH AREA ELEMENT
C     LYAREA(J) IS THE LENGTH OF THE J-TH AREA ALONG THE Y DIRECTION
           READ(10,*)XAREA(J),LXAREA(J),YAREA(J),LYAREA(J)
 160       CONTINUE
      ENDIF
 170  CONTINUE
C*************************************************************************
C     END OF DATA INPUT SECTION
C     BEGIN CALCULATION OF T(X,Y,Z)
C     THE SUBROUTINES USED IN THE CALCULATION ARE:
C     1) UZERO(N,M) - CALCULATES THE FOURIER COSINE TRANSFORM OF THE
C        FUNCTION, U(X,Y), THE POWER DENSITY FUNCTION FOR ALL OF THE
C        HEAT SOURCES.
C     2) FUN0(N,M) - CALCULATES THE FOURIER COEFFICIENTS FOR THE LAYERED
C        STRUCTURE USING THE THERMAL RECURSION RELATION
C*************************************************************************
      P04LK = 4.0 * P0 / ( LX* LY)
      PILX = PI /  LX
      PILY = PI /  LY
C     CALCULATE THE FOURIER COMPONENTS OF THE HEAT SOURCES, U(N,M)
C*************************************************************************
      DO 300 MM=1,MUP
      M = MM - 1
      DO 250 NN=1,NUP
         N = NN - 1
         ARUZER(NN,MM)=UZERO(N,M)
 250  CONTINUE
 300  CONTINUE
C*************************************************************************
C     END OF U(N,M) CALCULATION AND BEGINNING OF MAJOR LOOP FOR Z
```

```
C*****************************************************************************
C     CALCULATE THE FUN0(N,M) FROM THE RECURSION RELATION
C*****************************************************************************
      DO 400 MM=1,MUP
      M = MM - 1
      DO 350 NN=1,NUP
         N = NN - 1
         ARFUN0(NN,MM)=FUN0(N,M)*ARUZER(NN,MM)
 350  CONTINUE
 400  CONTINUE
C     THE FOLLOWING IF.THEN.ELSE CONSTRUCTION OPERATES ACCORDING TO THE
C     TYPE OF ANALYSIS TO BE USED.
C     FOR THE POINT FUNCTION ANALYSIS, THE 3000 LOOP IS USED
C     FOR THE LINE AVERAGE ANALYSIS, THE 4000 LOOP IS USED
C     FOR THE AREA AVERAGE ANALYSIS, THE 5000 LOOP IS USED
C     THIS PORTION DOES THE POINT FUNCTION CALCULATION
      IF (ITYPE.EQ.1) THEN
C     BEGINNING OF POINT FUNCTION ANALYSIS
      WRITE(12,2)
      WRITE(12,40)
      WRITE(12,43)
      DO 3000  IY=1,ILY
      DO 3100 MM=1,MUP
         M = MM - 1
         COSYT(MM)=COS(FLOAT(M)*Y(IY)*PILY)
 3100 CONTINUE
      DO 3000 IX=1,ILX
         SUM=0.0
         DO 3300 MM=1,MUP
         M = MM - 1
         DO 3200 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
            TOP = ARFUN0(NN,MM) * COS(FLOAT(N)*X(IX)*PILX) * COSYT(MM)
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
 3200    CONTINUE
 3300    CONTINUE
         TEMP = P04LK * SUM/K(NLAY)
         WRITE(12,44)X(IX),Y(IY),TEMP
 3000 CONTINUE
C     END OF POINT FUNCTION CALCULATION
      ELSE IF (ITYPE.EQ.2) THEN
C     THIS PORTION DOES THE LINE AVERAGE CALCULATION
      WRITE(12,2)
      WRITE(12,41)
      WRITE(12,45)
```

```
      DO 4000  IY=1,ILY
      DO 4010 MM=1,MUP
         M = MM - 1
         COSYT(MM)=COS(FLOAT(M)*Y(IY)*PILY)
 4010 CONTINUE
      DO 4000 J=1,NLINE
         SUM=0.0
         DO 4200 MM=1,MUP
         M = MM - 1
         DO 4100 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
          IF(N.EQ.0) GO TO 4160
           TERMX=SIN(FLOAT(N)*PI*(XLINE(J)+LXLINE(J))/LX)
     1           -SIN(FLOAT(N)*PI*XLINE(J)/LX)
           TERMX=TERMX*LX/(FLOAT(N)*PI)
           GO TO 4165
 4160  TERMX=LXLINE(J)
 4165  CONTINUE
            TOP = ARFUN0(NN,MM) * COSYT(MM) * TERMX
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
 4100    CONTINUE
         TEMP = P04LK * SUM / (LXLINE(J) * K(NLAY))
 4200    CONTINUE
      WRITE(12,46)J,XLINE(J),LXLINE(J),Y(IY),TEMP
 4000 CONTINUE
C     END OF LINE AVERAGE PORTION OF THE CODE
      ELSE IF (ITYPE.EQ.3) THEN
C     THIS PART PERFORMS THE AREA AVERAGE CALCULATION
      WRITE(12,2)
      WRITE(12,42)
      WRITE(12,47)
      DO 5000 J=1,NAREA
         SUM=0.0
         DO 5100 MM=1,MUP
         M = MM - 1
         DO 5200 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
      AREA=0.0
         IF(N.EQ.0) GO TO 5160
           TERMX = SIN(FLOAT(N)*PI*(XAREA(J) +  LXAREA(J))/ LX)
     1           - SIN(FLOAT(N)*PI*XAREA(J)/ LX)
           TERMX=TERMX* LX/(FLOAT(N)*PI)
           GO TO 5165
```

```
5160      TERMX= LXAREA(J)
5165      IF(M.EQ.0) GO TO 5164
           TERMY = SIN(FLOAT(M)*PI*(YAREA(J) +  LYAREA(J))/ LY)
     1             - SIN(FLOAT(M)*PI*YAREA(J)/ LY)
           TERMY=TERMY* LY/(FLOAT(M)*PI)
           GO TO 5166
5164      TERMY= LYAREA(J)
5166      TERMI=TERMX*TERMY
         AREA=TERMI
            TOP = ARFUN0(NN,MM) * AREA
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
5200     CONTINUE
         TEMP = P04LK * SUM / (LXAREA(J)*LYAREA(J)*K(NLAY))
5100     CONTINUE
      WRITE(12,48)J,XAREA(J),LXAREA(J),YAREA(J),LYAREA(J),TEMP
5000  CONTINUE
      END IF
      WRITE(12,27) LX, LY
      WRITE(12,4) NLAY
      WRITE(12,3)
      DO 3001 I=NLAY,1,-1
      WRITE(12,5) I,L(I),I,K(I)
3001  CONTINUE
      WRITE(12,6)NUP,MUP
      WRITE(12,11) P0
      WRITE(12,7)NSOUR
      WRITE(12,8)
      WRITE(12,9)
      DO 3888 I=1,NSOUR
      WRITE(12,10)I,WTSOUR(I),XSOUR(1),YSOUR(I), LXSOUR(I), LYSOUR(I)
3888  CONTINUE
      IF(ITYPE.EQ.1) THEN
          GOTO 7000
      ELSE IF (ITYPE.EQ.2) THEN
          WRITE(12,32)NLINE
          DO 6100 J=1,NLINE
          WRITE(12,33)J,XLINE(J),LXLINE(J)
6100  CONTINUE
      ELSE IF (ITYPE.EQ.3) THEN
          WRITE(12,34) NAREA
          DO 6200 J=1,NAREA
          WRITE(12,35)J,XAREA(J),LXAREA(J),YAREA(J),LYAREA(J)
6200  CONTINUE
      END IF
      GO TO 7000
3999  WRITE(12,88)
7000  STOP
      END
C*******************************************************************************
C     END OF THE MAIN PROGRAM
```

```
C     FUNCTION UZERO LISTING
C*************************************************************************
C     DESCRIPTION OF THE FUNCTION UZERO(N,M)
C     THIS FUNCTION CALCULATES THE DOUBLE FOURIER COSINE TRANSFORM
C     OF THE POWER DENSITY FUNCTION, U(X,Y).  THIS IS THE TRANSFORM
C     FOR ALL OF THE HEAT SOURCES.  THE ASSUMPTION IS MADE THAT THE
C     POWER DENSITY IS UNIFORM AND EQUAL TO UNITY OVER THE SURFACE
C     OF THE HEATING ELEMENTS.  THAT IS,
C       U(X,Y)=1  (XSOUR(I)<=X<=XSOUR(I)+LXSOUR(I)  AND
C                  YSOUR(I)<=Y<=YXOUR(I)+LYSOUR(I) ).
C       U(X,Y)=0  OTHERWISE.
C     UNDER THESE CONDITIONS, IT IS POSSIBLE TO ANALYTICALLY EVALUATE
C     THE DOUBLE INTEGRAL FOR EACH HEATING ELEMENT.  AS THE HEATING
C     ELEMENTS ARE ASSUMED TO BE INDEPENDENT, THE CONTRIBUTION FROM
C     EACH ELEMENT MAY BE ADDED TO OBTAIN THE U(N,M) FOR ALL.
C
C     NONUNIFORM POWER DENSITIES MAY BE TAKEN CARE OF BY USING THE
C     WEIGHTING FACTOR FOR EACH ELEMENT (READ IN THE MAIN PROGRAM)
C*************************************************************************
      FUNCTION UZERO(N,M)
      DIMENSION WTSOUR(50), XSOUR(50), YSOUR(50)
      REAL LXSOUR(50), LYSOUR(50), K(20), LX, LY, L(20)
      COMMON  LX, LY, NLAY, K, L
      COMMON NSOUR,WTSOUR,XSOUR,YSOUR, LXSOUR, LYSOUR
      PI=3.14159265
      UZERO=0.0
      DO 500 I=1,NSOUR
         IF(N.EQ.0) GO TO 100
           TERMX = SIN(FLOAT(N)*PI*(XSOUR(I) +  LXSOUR(I))/ LX)
     1             - SIN(FLOAT(N)*PI*XSOUR(I)/ LX)
           TERMX=TERMX* LX/(FLOAT(N)*PI)
           GO TO 150
 100     TERMX= LXSOUR(I)
 150     IF(M.EQ.0) GO TO 200
           TERMY = SIN(FLOAT(M)*PI*(YSOUR(I) +  LYSOUR(I))/ LY)
     1             - SIN(FLOAT(M)*PI*YSOUR(I)/ LY)
           TERMY=TERMY* LY/(FLOAT(M)*PI)
           GO TO 250
 200     TERMY= LYSOUR(I)
 250     TERMI=TERMX*TERMY
         UZERO=UZERO+TERMI*WTSOUR(I)
 500  CONTINUE
      RETURN
      END
C*************************************************************************
```

```
C     FUNCTION FUN0 LISTING
C
C     DESCRIPTION OF THE FUNCTION FUN0(N,M)
C
C*************************************************************************
      FUNCTION FUN0(N,M)
      DIMENSION WTSOUR(50), XSOUR(50), YSOUR(50)
      REAL LXSOUR(50), LYSOUR(50), K(20), LX, LY, L(20)
      COMMON  LX, LY, NLAY, K, L
      COMMON NSOUR,WTSOUR,XSOUR,YSOUR, LXSOUR, LYSOUR
      PI=3.14159265
      GAMMA=SQRT((FLOAT(N)*PI/ LX)**2 + (FLOAT(M)*PI/ LY)**2)
      IF (GAMMA.EQ.0) GO TO 120
C     FOR GAMMA.GT.0, START WITH BOTTOM LAYER
      FUN0=TANH(GAMMA*L(1))
      IF (NLAY.EQ.1) GO TO 110
C     THE 100 LOOP DOES THE RECURSION RELATION FOR NONZERO GAMMA
      DO 100 I=2,NLAY
      TOP=FUN0*K(I)+K(I-1)*TANH(GAMMA*L(I))
      BOT=K(I-1)+K(I)*FUN0*TANH(GAMMA*L(I))
      FUN0=TOP/BOT
 100  CONTINUE
 110  FUN0=FUN0/GAMMA
      RETURN
 120  FUN0=L(1)
      IF (NLAY.EQ.1) GO TO 130
C     THE 125 LOOP DOES THE RECURSION RELATION FOR GAMMA EQUAL TO ZERO
      DO 125 I=2,NLAY
      FUN0=(FUN0*K(I)+K(I-1)*L(I))/K(I-1)
 125  CONTINUE
 130  RETURN
      END
```

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